

Unfoldings of saddle-nodes and their Dulac time [☆]P. Mardešić ^a, D. Marín ^b, M. Saavedra ^c, J. Villadelprat ^{d,*}^a *Institut de Mathématiques de Bourgogne, UFR Sciences et Techniques, Université de Bourgogne, B.P. 47870, 21078 Dijon, France*^b *Departament de Matemàtiques, Facultat de Ciències, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain*^c *Departamento de Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Barrio Universitario, Concepción, Casilla 160-C, Chile*^d *Departament d'Enginyeria Informàtica i Matemàtiques, ETSE, Universitat Rovira i Virgili, 43007 Tarragona, Spain*

Received 26 April 2015; revised 18 March 2016

Available online 13 September 2016

Abstract

In this paper we study unfoldings of saddle-nodes and their Dulac time. By unfolding a saddle-node, saddles and nodes appear. In the first result ([Theorem A](#)) we give a uniform asymptotic expansion of the trajectories arriving at the node. Uniformity is with respect to all parameters including the unfolding parameter bringing the node to a saddle-node and a parameter belonging to a space of functions. In the second part, we apply this first result for proving a regularity result ([Theorem B](#)) on the Dulac time (time of Dulac map) of an unfolding of a saddle-node. This result is a building block in the study of bifurcations of critical periods in a neighborhood of a polycycle. Finally, we apply [Theorems A and B](#) to the study of critical periods of the Loud family of quadratic centers and we prove that no bifurcation occurs for certain values of the parameters ([Theorem C](#)).

© 2016 Elsevier Inc. All rights reserved.

MSC: 34C07

Keywords: Period function; Unfolding of a saddle-node; Asymptotic expansions

[☆] This work was partially supported by the FONDECYT project 1120333, the grants MTM2011-26674-C02-01, MTM-2008-03437 from MINECO/FEDER, UNAB10-4E-378, co-funded by ERDF “A way to build Europe” and by the French ANR-11-BS01-0009 STAAVF.

* Corresponding author.

E-mail address: jordi.villadelprat@urv.cat (J. Villadelprat).