

On the time function of the Dulac map for families of meromorphic vector fields

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Received 3 July 2002, in final form 16 January 2003

Published 7 March 2003

Online at stacks.iop.org/Non/16/855

Recommended by L Bunimovich

Abstract

Given an analytic family of vector fields in \mathbb{R}^2 having a saddle point, we study the asymptotic development of the time function along the union of the two separatrices. We obtain a result (depending uniformly on the parameters) which we apply to investigate the bifurcation of critical periods of quadratic centres.

Mathematics Subject Classification: 34C07, 34C23, 34C25

1. Introduction and setting of the problem

The aim of this paper is to calculate the first terms in the development of the time function of the passage around a saddle point of a family of meromorphic vector fields. A good scale permitting a uniform development (with respect to the parameter) involves the compensator function of Ecalle–Roussarie. The study was motivated by the necessity of such a result for the investigation of the bifurcation diagram of the period function of quadratic centres. Let $\{X_\mu, \mu \in \Lambda\}$ be the family of quadratic vector fields having a centre at the origin (parametrized by means of the coefficients), the problem is to decompose the parameter space as $\Lambda = \bigcup V_i$ in such a way that if μ_1 and μ_2 belong to the same set V_i then the period function of the centre of X_{μ_1} and the one of X_{μ_2} are qualitatively the same. With this we mean that their critical periods are equal in number, character (minimum or maximum) and distribution.

⁴ Supported by the Conseil Régional de Bourgogne under the grant 01-5143-27-00021.

⁵ Supported through a European Community Marie Curie Fellowship.