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UNFOLDING OF RESONANT SADDLES AND THE DULAC TIME

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Abstract. In this work we study unfoldings of planar vector fields in a neighbourhood of a resonant saddle. We give a $\mathcal{C}^k$ normal form for the unfolding with respect to the conjugacy relation. Using our normal form we determine an asymptotic development, uniform with respect to the parameters, of the Dulac time of a resonant saddle deformation. Conjugacy relation instead of weaker equivalence relation is necessary when studying the time function. The Dulac time of a resonant saddle can be seen as the basic building block of the total period function of an unfolding of a hyperbolic polycycle.

Introduction. In this work we study unfoldings of planar vector fields in a neighbourhood of a resonant saddle. We give a $\mathcal{C}^k$ normal form for the unfolding with respect to the conjugacy relation. This generalizes the known orbital normal form with respect to the equivalence relation [4] and [13].

Using our normal form we determine an asymptotic development, uniform with respect to the parameters, for the Dulac time of a resonant saddle. Our asymptotic development of the Dulac time is of a similar nature as the asymptotic expansion of the Dulac map given in [13]. It generalizes our previous work [7] dealing with the Dulac time of orbitally linearizable families, but without being as explicit on the coefficients.

Our initial motivation was the problem of finite "cyclicity" (i.e., existence of a local uniform bound) for the number of critical points of the period function of polynomial vector fields on hyperbolic or more general polycyles. The condition of non-criticality of the period appears for instance in the bifurcation theory of subharmonics. Under the non-criticality of the period, zeros of appropriate Melnikov functions guarantee the persistence of a subharmonic periodic orbit of a Hamiltonian under a periodic non-autonomous deformation (see Theorem 4.62 of [3]).

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