Journal of Dynamical and Control Systems
https://doi.org/10.1007/s10883-020-09520-3

Bifurcations of Zeros in Translated Families of Functions and Applications

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Received: 8 July 2020 / Revised: 26 August 2020 / Accepted: 11 October 2020 /
Published online: 09 November 2020

Abstract
In this paper, we study the creation of zeros in a certain type of families of functions. The families studied are given by the difference of two basic functions with a translation made in the argument of one of these functions. The problem is motivated by applications in the 16th Hilbert problem and its ramifications. Here, we apply the results obtained to the study of bifurcations of critical periods in the Loud family of quadratic centers.

Keywords Critical period · Bifurcations · Cyclicity · Centers

Mathematics Subject Classification (2010) 34C05 (34C23 34C25 37G15)

1 Introduction and Main Results

This paper is motivated by the study of bifurcations of critical points of the period function in a neighborhood of a polycycle. A key problem in these studies is the breaking of separatrices of the polycycle. It appears also in the study of limit cycles corresponding to fixed points of the Poincaré return map of a family of planar vector fields. Contrary to the situation in the study of limit cycles, here by breaking a polycycle it is replaced by a polycycle with less vertices. The simplest situation is when a polycycle with two vertices is broken and a saddle loop polycycle is created.

The cyclicity (i.e., number of limit cycles appearing by perturbation) of hyperbolic polycycles has been extensively studied in [1, 4, 15–18, 20, 24] among others.

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