Afternotes on PHM: Harmonic ENO Methods

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Summary. PHM methods have been used successfully as reconstruction procedures to design high-order Riemann solvers for nonlinear scalar and systems of conservation laws, (see [8], [1], [4]). We introduce a new class of polynomial reconstruction procedures based on the harmonic mean of the absolute values of finite differences used as difference-limiter, following the original idea used before to design the piecewise hyperbolic method, introduced in [8]. We call those methods 'harmonic ENO methods', (HENO). Furthermore, we give analytical and numerical evidence of the good behavior of these methods used as reconstruction procedures for the numerical approximation by means of shock-capturing methods for scalar and systems of conservation laws in 1D. We discuss, in particular, the behavior of a fourth order harmonic ENO method,(HENO4 in short), compared with PHM, ENO3 and WENO5 methods, (see [2], [10], [3]).

1 Introduction

In this paper, we shall consider numerical approximations to nonlinear conservation laws of the form:

$$\frac{\partial \mathbf{u}}{\partial t} + \sum_{i=1}^{d} \frac{\partial \mathbf{f}_i(\mathbf{u})}{\partial x_i} = 0 \tag{1}$$

where $\mathbf{u}$ is a m-dimensional vector of unknowns and $\mathbf{f}_i(\mathbf{u})$ are d vector-valued functions called *fluxes*, We assume strong hyperbolicity, i.e. the Jacobian matrices of the former system

$$\mathbf{A}_i = \frac{\partial \mathbf{f}_i(\mathbf{u})}{\partial \mathbf{u}} \tag{2}$$

locally diagonalize with real eigenvalues and a complete system of eigenvectors. The one-dimensional case of the system will be in the form

$$\mathbf{u}_t + (\mathbf{f}(\mathbf{u}))_x = 0 \tag{3}$$

with the initial value condition

$$\mathbf{u}(x, 0) = \mathbf{u}_0(x) \tag{4}$$