PAMM · Proc. Appl. Math. Mech. **7**, 1024101–1024102 (2007) / **DOI** 10.1002/pamm.200700672

# Shock-capturing schemes: high accuracy versus total-variation boundedness

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In this reseach work we analyze the total variation growth of some high order accurate reconstruction procedures used for the design of shock capturing schemes. This study allows to measure how oscillatory a high order accurate method is in terms of the basic elementary function chosen to increase the order of accuracy.



## 1 Introduction

Shock capturing schemes that approximate the physical consistent solutions of conservation laws are total variation bounded ( [2]). The complex structure of these solutions exhibit jump discontinuities (shocks and contacts) and fine structure. High order accuracy in smooth regions of the solution and sharp profiles of jump discontinuities while avoiding spurious oscillations are demanding and desirable features of shock capturing schemes.

The design of high order accurate numerical schemes maintaining bounded the total variation of the solution is a challenging goal. High order accuracy in space of numerical schemes is obtained by extrapolating at cell interfaces piecewise smooth functions that reconstruct the solution up to an order of accuracy. In this research work we analyze the behavior of some high order accurate shock capturing schemes with respect to their total variation boundedness.

## 2 Local variation of basic reconstructing functions

We focus on the numerical approximations of a piecewise smooth function $g(x)$ from its cell averages. We define $v_j$ at a uniform grid of stepsize $h > 0$, $x_j = jh$, through the conservation property as

$$v_j = \frac{1}{h}\int_{x_{j-\frac{1}{2}}}^{x_{j+\frac{1}{2}}} g(x,t_n)dx \tag{1}$$

The strategy to achieve high order consists of capturing subscales of the original chosen scale by means of a piecewise smooth function, $R$, such that the restriction to each computational cell $[x_{j-\frac{1}{2}}, x_{j+\frac{1}{2}}]$ is a suitable elementary function, $R_j$, such that the conservation property (1) is satisfied

$$v_j = \frac{1}{h}\int_{x_{j-\frac{1}{2}}}^{x_{j+\frac{1}{2}}} R_j(x,t_n)dx \tag{2}$$

approximating $g$ up to a degree of accuracy. We restrict our study to third order accurate piecewise smooth functions, $R$, such that each $R_j$ is determined from the conservation property (2) and two more conditions usually obtained from differences of $v_j$. We denote by $d_{j+\frac{1}{2}} = \frac{v_{j+1}-v_j}{h}$. We use the following two interpolating conditions to ensure third order accuracy in smooth regions

$$R'_j(x_j - \frac{h}{2}) = d_{j-\frac{1}{2}} \tag{3}$$

$$R'_j(x_j + \frac{h}{2}) = d_{j+\frac{1}{2}} \tag{4}$$

The local variation bounded property of piecewise smooth reconstructing functions was introduced in ( [5]) as a necessary condition to ensure the total variation boundedness of a scheme.

We consider three main basic reconstructing functions namely: parabolas, hyperbolas and logarithmic functions. These basic functions have been used for the design of high order accurate schemes in [1, 3–7] among others. We represent by $p_j, r_j$

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