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The period function of generalized Loud's centers ☆

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ABSTRACT

In this paper a three parameter family of planar differential systems with homogeneous nonlinearities of arbitrary odd degree is studied. This family is an extension to higher degree of Loud's systems. The origin is a nondegenerate center for all values of the parameter and we are interested in the qualitative properties of its period function. We study the bifurcation diagram of this function focusing our attention on the bifurcations occurring at the polycycle that bounds the period annulus of the center. Moreover we determine some regions in the parameter space for which the corresponding period function is monotonous or it has at least one critical period, giving also its character (maximum or minimum). Finally we propose a complete conjectural bifurcation diagram of the period function of these generalized Loud's centers.

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1. Introduction and statement of the results

The present paper deals with planar polynomial ordinary differential systems and we study the qualitative properties of the period function of centers. Recall that a critical point p of a planar differential system is a *center* if it has a punctured neighborhood that consists entirely of periodic orbits surrounding p. The largest neighborhood with this property is called the *period annulus* of the center and in what follows it will be denoted by $\mathscr{P}$. The *period function* assigns to each periodic orbit in $\mathscr{P}$ its period. If the period function is constant then the center is called *isochronous*. The study of the period function is a nontrivial problem and questions related to its behavior have been extensively

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