Contents lists available at ScienceDirect



Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

# Uniqueness of limit cycles for sewing planar piecewise linear systems



CrossMark

霐

## João C. Medrado<sup>a,\*</sup>, Joan Torregrosa<sup>b</sup>

<sup>a</sup> Instituto de Matemática e Estatística, Universidade Federal de Goiás, 74001-970 Goiânia, Goiás, Brazil
<sup>b</sup> Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain

#### ARTICLE INFO

Article history: Received 3 January 2015 Available online 3 June 2015 Submitted by W. Sarlet

Keywords: Non-smooth differential system Uniqueness of limit cycle Piecewise sewing planar linear differential system

#### ABSTRACT

This paper proves the uniqueness of limit cycles for sewing planar piecewise linear systems with two zones separated by a straight line,  $\Sigma$ , and only one  $\Sigma$ -singularity of monodromic type. The proofs are based in an extension of Rolle's Theorem for dynamical systems on the plane.

@ 2015 Elsevier Inc. All rights reserved.

### 1. Introduction and main results

Usually the models used in many problems related to engineering, biology, control theory, design of electric circuits, mechanical systems, economics science, and medicine are differential systems that are neither analytic, nor differentiable. A good tool to describe the dynamics of these models is the study of piecewise differential systems. See [1,8] for a wide selection of models and real applications. Typically this class of systems is obtained using two or more linear vector fields that are defined on different regions separated by discontinuity boundaries. In particular, a circuit having an ideal switch with state feedback can be modeled with a planar piecewise linear system where the discontinuity boundary is defined by a straight line, see Sec. 1.1.7 of [1].

Planar linear differential systems are completely understood using only linear algebra and they do not present isolated periodic orbits, so-called *limit cycles*. This is not the case for piecewise linear differential systems. The classification of the different phase portraits or the study of the maximum number of limit cycles are still open problems, even when the number of regions is small, two in our case, or the boundaries are straight lines. The existence of real and/or virtual singularities, connection of separatrices, isolated periodic orbits, ... increase, in comparison with the linear one, the number of possible phase portraits in the

\* Corresponding author. E-mail addresses: medrado@ufg.br (J.C. Medrado), torre@mat.uab.cat (J. Torregrosa).

http://dx.doi.org/10.1016/j.jmaa.2015.05.064 0022-247X/© 2015 Elsevier Inc. All rights reserved.