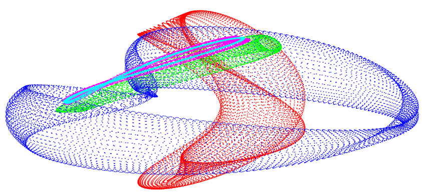
Contribution to the Study of Fourier Methods for Quasi–Periodic Functions and the Vicinity of the Collinear Libration Points

José María Mondelo

Departament de Matemàtica Aplicada i Anàlisi
Universitat de Barcelona



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Certifico que la present memòria ha estat
realitzada per José María Mondelo González
i dirigida per mi.

Barcelona, 23 de maig de 2001

Gerard Gómez i Muntané

A mi familia.

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Preface

This work has been organized in three parts. The first two ones contain the main results, and the last one, which has been divided in several appendices, has complementary results.

The first part of the work (Chapters 1 to 5) is dedicated to the development and study of a procedure for the accurate computation of frequencies, as well as the related Fourier coefficients, of a quasi–periodic function, using as only input input a equally–spaced sampling of the function to be analyzed over a finite time interval.

The first technique for the accurate determination of frequencies has been introduced by J. Laskar ([17], [19], [18]). It is based on the maximization of the formula that gives the Fourier coefficients of a function with respect to the harmonic index, but taking it as a real number. This procedure has been applied to the study of the long–term dynamics of the Solar System ([17]), as well as to the study of chemistry and particle accelerator models through the computation of *frequency maps* ([18]). Some methodology for frequency determination has also been introduced in [12],[13],[10],[11]. In these works, the determination of frequencies has been applied to development of semi–analytical models for the motion in the Solar System.

Our procedure takes the methodology developed in [12],[13],[10][11] as a starting point. It is based in asking for equality between the Discrete Fourier Transform (DFT) of the analyzed function and its quasi–periodic approximation. Error estimates are obtained and illustrated with numerical examples. Also, in the line of the previously–mentioned works, we apply our procedure to the development of simplified models for the motion in the Solar System.

The second part of the work (Chapters 6 to 7) is devoted to the study to the dynamics in the vicinity of the collinear equilibrium points of the three–dimensional Restricted Three–Body Problem (RTBP) for the Earth–Moon mass parameter.

The first systematic study of this vicinity has been done in [10] and [**?**], using as a tool the reduction to the central manifold of the collinear equilibrium points. This is a semi–analytical technique, which limits the region that can be explored by the convergence of the expansions computed. The same methodology has also been applied to the study of the collinear equilibrium points of a model for the Earth–Moon system, called the Quasi–Bicircular Problem ([3]). In this last study, the convergence constraints are still more severe.

In this work, we follow the families of periodic orbits and invariant 2D tori of the center manifolds of the three collinear libration points using purely numerical procedures. With this approach, we can extend the analysis of the phase space done in [10] and [**?**] to a wider range of energy values, that now include several bifurcations, and also to the L_3 libration point. The methodology used for the continuation of invariant tori is based in [7], with some modifications in order to account for variable excitations and some additional parameters needed for our exploration. We have followed parallel strategies in order to cope with the large amount of computations required. They have been carried out on HIDRA, one of the Beowulf clusters of the Barcelona Dynamical Systems Group.

The third and last part of this report consists in several appendices, which give some additional results that have been taken apart from the main text in order to improve its readability.

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