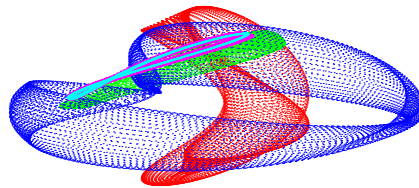


Contribution to the Study of Fourier Methods for Quasi-Periodic Functions and the Vicinity of the Collinear Libration Points

José María Mondelo

Departament de Matemàtica Aplicada i Anàlisi
Universitat de Barcelona



Programa de doctorat de Matemàtica Aplicada i Anàlisi.
Bienni 1996–98.

Memòria presentada per a aspirar al grau de
Doctor en Matemàtiques per la Universitat
de Barcelona

Certifico que la present memòria ha estat
realitzada per José María Mondelo González
i dirigida per mi.

Barcelona, 23 de maig de 2001

Gerard Gómez i Muntané

A mi familia.

Acknowledgements

I would like to thank my thesis advisor, G. Gómez, for the many hours we have spent together, and for his support and advice, which has never been just academic. I am also indebted with C. Simó, who has always been aware of this work and has given many hints and ideas that have been critical for its developement. I would like to thank also À. Jorba, for several suggestions and for introducing me in the world of parallel computing. J. Masdemont has provided me software to acces the JPL ephemeris and to evaluate the c_i functions of Appendix A. I would like to mention also J. Font and J. Timoneda, who have always provided me one of the most powerful computing enviornments of the Department. They have also shared with me their expertise in hardware and system management, which has allowed me to get my current job. All my colleagues of the Department of Applied Mathematics and Analysis of the University of Barcelona have contributed to provide a very friendly environment to work in. I would like to especially mention A. González, F. Naselli and J. Puig, for their friendship and support during my last year there. And finally, I would like to thank my family. Whithout their support, this work would never have been done.

Preface

This work has been organized in three parts. The first two ones contain the main results, and the last one, which has been divided in several appendices, has complementary results.

The first part of the work (Chapters 1 to 5) is dedicated to the development and study of a procedure for the accurate computation of frequencies, as well as the related Fourier coefficients, of a quasi-periodic function, using as only input a equally-spaced sampling of the function to be analyzed over a finite time interval.

The first technique for the accurate determination of frequencies has been introduced by J. Laskar ([17], [19], [18]). It is based on the maximization of the formula that gives the Fourier coefficients of a function with respect to the harmonic index, but taking it as a real number. This procedure has been applied to the study of the long-term dynamics of the Solar System ([17]), as well as to the study of chemistry and particle accelerator models through the computation of *frequency maps* ([18]). Some methodology for frequency determination has also been introduced in [12],[13],[10],[11]. In these works, the determination of frequencies has been applied to development of semi-analytical models for the motion in the Solar System.

Our procedure takes the methodology developed in [12],[13],[10],[11] as a starting point. It is based in asking for equality between the Discrete Fourier Transform (DFT) of the analyzed function and its quasi-periodic approximation. Error estimates are obtained and illustrated with numerical examples. Also, in the line of the previously-mentioned works, we apply our procedure to the development of simplified models for the motion in the Solar System.

The second part of the work (Chapters 6 to 7) is devoted to the study to the dynamics in the vicinity of the collinear equilibrium points of the three-dimensional Restricted Three-Body Problem (RTBP) for the Earth-Moon mass parameter.

The first systematic study of this vicinity has been done in [10] and [?], using as a tool the reduction to the central manifold of the collinear equilibrium points. This is a semi-analytical technique, which limits the region that can be explored by the convergence of the expansions computed. The same methodology has also been applied to the study of the collinear equilibrium points of a model for the Earth-Moon system, called the Quasi-Bicircular Problem ([3]). In this last study, the convergence constraints are still more severe.

In this work, we follow the families of periodic orbits and invariant 2D tori of the center manifolds of the three collinear libration points using purely numerical procedures. With this approach, we can extend the analysis of the phase space done in [10] and [?] to a wider range of energy values, that now include several bifurcations, and also to the L_3 libration point. The methodology used for the continuation of invariant tori is based in [7],

with some modifications in order to account for variable excitations and some additional parameters needed for our exploration. We have followed parallel strategies in order to cope with the large amount of computations required. They have been carried out on HIDRA, one of the Beowulf clusters of the Barcelona Dynamical Systems Group.

The third and last part of this report consists in several appendices, which give some additional results that have been taken apart from the main text in order to improve its readability.

Contents

Acknowledgements	v
Preface	vii
I Numerical Fourier analysis of quasi-periodic functions	1
1 The Discrete Fourier Transform (DFT)	5
1.1 Preliminaries and notation	5
1.2 Leakage effect and filtering	7
1.3 Aliasing effect	11
2 Procedures for the refined Fourier analysis	13
2.1 Introduction	13
2.2 First approximation of frequencies	14
2.3 Computation of the amplitudes	15
2.4 Improvement of frequencies and amplitudes	17
2.5 Implementation details	18
2.5.1 Algorithm for the procedure	18
2.5.2 Use of trigonometric recurrences	19
2.5.3 Evaluation of the DFT of sines and cosines	20
3 Error estimates	25
3.1 Introduction and notation	25
3.2 Error bounds for $\ Dg(y)^{-1}\ _\infty$	26
3.2.1 Error estimation for known frequencies	31
3.2.2 General case	35
3.3 Error bounds for $\ \Delta b\ _\infty$	42
3.4 Final results	49
4 A numerical example	53
4.1 The family of functions analyzed	53
4.2 Numerical results	53

5	Development of Solar System models	61
5.1	Introduction	61
5.2	Fourier analysis	63
5.2.1	Fourier analysis of the c_i functions	63
5.2.2	Fourier analysis of the positions of the planets	67
5.3	Generation of simplified Solar System models	74
5.3.1	Adjustment by linear combinations of basic frequencies	74
5.3.2	Simplified models for the Earth–Moon case	75
5.3.3	Simplified models for the Sun–Earth+Moon case	83
II	The neighborhood of the collinear equilibrium points in the RTBP	89
6	Methodology	93
6.1	Refinement and continuation of periodic orbits	93
6.1.1	The system of equations	93
6.1.2	Refinement of a periodic orbit	94
6.1.3	Continuation of a family of p.o.	95
6.2	Refinement and continuation of invariant tori	96
6.2.1	Indeterminations of the Fourier representation	98
6.2.2	Multiple shooting	99
6.2.3	The system of equations	99
6.2.4	Refinement of an invariant torus	99
6.2.5	Continuation of a family of tori	100
6.2.6	Error estimation	101
6.3	Starting from a periodic orbit	101
6.3.1	Starting “longitudinally” to the periodic orbit	103
6.3.2	Starting “transversally” to the periodic orbit	104
6.4	Computational aspects	104
6.4.1	Continuation of a 1–parametric family of tori	104
6.4.2	On the computing effort	105
6.4.3	Computation of kernel and minimum–norm corrections using QR with column pivoting	106
6.4.4	Parallel strategies	107
7	Numerical Results	109
7.1	Lyapunov families	109
7.2	Halo–type orbits	115
7.3	Families of invariant tori	121
7.3.1	Invariant tori starting around vertical orbits	121
7.3.2	Invariant tori starting around halo and halo–type orbits	124
7.4	Summary of results	125
7.5	Additional families of invariant tori	131

III	Appendices	135
A	Models of motion in the Solar System	139
A.1	The Restricted Three Body Problem (RTBP)	139
A.2	The Bicircular Problem (BCP)	141
A.3	The Quasi-Bicircular problem (QBCP)	142
A.4	The Solar System as RTBP+perturbations	143
A.4.1	Deduction of the equations	143
B	The discontinuities of the JPL ephemeris	149
B.1	Structure of JPL's ephemeris files	149
B.2	Jump discontinuities corresponding to DE406	149
C	Fourier expansions	151
C.1	Notation	151
C.2	Expansions of the c_i functions, Earth-Moon case	151
C.3	Solar System bodies, Earth-Moon case	165
C.4	Expansions of the c_i functions, Sun-Earth+Moon case	181
C.5	Solar System bodies, Sun-Earth+Moon case	186
D	Evolution of the families of periodic orbits	191
D.1	Lyapunov families	191
D.1.1	Planar Lyapunov families	194
D.2	Halo and halo-type families	198
D.2.1	Halo families	198
D.2.2	Period-duplicated halo families	202
D.2.3	Period-triplicated halo families	207
E	Resum	221
E.1	Anàlisi de Fourier de funcions quasiperiòdiques	222
E.1.1	Notacions per la DFT	222
E.1.2	Primera aproximació de freqüències	223
E.1.3	Càlcul de les amplituds suposant freqüències conegudes	223
E.1.4	Refinament conjunt de freqüències i amplituds	223
E.1.5	Algorisme	225
E.1.6	Estudi de l'error	226
E.1.7	Un exemple numèric	229
E.1.8	Aplicació al desenvolupament de models simplificats de moviment al Sistema Solar	231
E.2	L'entorn dels punts de libració colineals	238
E.2.1	Metodologia	238
E.2.2	Resultats numèrics	241