## Ballistic Transfers Across the 1:1 Resonance Around Vesta Following Invariant Manifolds

Josep-Maria Mondelo\*

Universitat Autònoma de Barcelona, Bellaterra 08193, Barcelona, Spain

Stephen B. Broschart<sup>†</sup>

California Institute of Technology, Pasadena, California 91109

and

Benjamin F. Villac<sup>‡</sup>

University of California, Irvine, Irvine, California 92697

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Motivated by the challenging crossing of the 1:1 resonance planned for the Dawn mission a general transfer design strategy is developed using the manifold structure of near-synchronous, unstable periodic orbits. Two families of transfers across the 1:1 resonance at the asteroid Vesta demonstrate the approach. A family of low-inclination low-energy transfers follows the stable and unstable manifolds of the libration orbits near Vesta's equilibrium points (in a surface-fixed rotating frame). A high-inclination family of transfers arises similarly from the manifold structure of near-polar near-circular orbits. Ballistic resonance crossing transfers near Vesta are presented and characterized. This transfer design methodology applies well to any solar system body where the dynamics are dominated by gravitational acceleration from a nonspherical central-body potential.

			$\tilde{\mathbf{r}} = (x, y, z)$	=	spacecraft position vector in body-fixed,
		Nomenclature	Speed	_	set of sigenvalues of matrix A
Ē Š	=	normalized spherical harmonic coefficients	SpecA	=	Set of eigenvalues of matrix A
		of the gravitational field	υ, μ	=	and parameter
$D\phi_T(X_0)$	=	monodromy matrix (state transition matrix after one period) of a <i>T</i> periodic orbit with	$V^s, V^u$	=	stable and unstable eigenvectors,
		initial condition V			respectively
		initial condition $\mathbf{A}_0$	$\mathbf{X} = (\mathbf{r}, \mathbf{p})$	=	spacecraft position and momenta in
$\mathbf{F}(\mathbf{X})$	=	associated (Hamiltonian) vector field			rotating, body-fixed Vesta centered frame
$H(\mathbf{r},\mathbf{p})$	=	orbiter Hamiltonian, also referred to as	$\mathbf{Z}(h)$	=	zero-velocity surface
		energy	$\kappa(\mathbf{r},h)$	=	implicit function defining the zero-velocity
h	=	arbitrary fixed value of the Hamiltonian			surfaces and allowable region of motion
		(energy)	$\lambda^{j}$	=	real part of <i>i</i> th eigenvalue
$i, j, k, k_1, k_2, m, n$	=	arbitrary integers	$\xi^s, \xi^u$	=	displacements along the stable and
$J(\tilde{\mathbf{r}}, \tilde{\mathbf{r}})$	=	first integral of motion for the Vesta orbiter			unstable eigenvectors, respectively
		dynamics in terms of position and velocity	$\phi_{i}, \theta$	=	t advance mapping along a trajectory and
$P_n$	=	Legendre polynomials	<i>T</i> 1, -		phase along a periodic orbit
$P_{\rm nm}, \bar{P}_{\rm nm}$	=	associated Legendre and normalized	$\psi(\theta, \xi^u, \xi^s)$	=	parametrization of a stable — unstable
		associated Legendre functions	1 ( ) 5 / 5 /		manifold
$P_1, P_2, P_3, P_4$	=	relative equilibria of the orbiter dynamics	$\boldsymbol{\omega}, \boldsymbol{\omega}$	=	angular velocity vector of Vesta (relative to
$\tilde{\mathbf{p}} = (p_x, p_y, p_z)$	=	orbiter momenta in rotating, body-fixed			inertial space) and associated magnitude
		Vesta centered frame	$\omega_p^j$	=	imaginary part of <i>j</i> th "in-plane" eigenvalue
$R_V$	=	radius of the reference sphere associated	Ĩ		(with small z, $p_z$ components)
		with the spherical harmonic gravity field	$\omega_v^j$	=	imaginary part of <i>i</i> th "out-of-plane"
$(r, \phi, \lambda)$	=	radius, latitude and longitude of spacecraft	-		eigenvalue (with large z, $p_z$ components)
· · · · ·		in the Vesta centered, body-fixed frame			C ( , , , , , , , , , , , , , , , , , ,

I. Introduction

A SOF September 2011, NASA's Dawn mission has successfully inserted into orbit around the asteroid Vesta. Dawn's Vesta exploration plan calls for eventual transfer to the low-altitude mapping orbit (LAMO) at ~460 km radius [1,2]. The LAMO has an orbital period smaller than Vesta's rotation period, so that the spacecraft must cross the 1:1 resonance between the spacecraft orbit period and Vesta's rotation period using its low-thrust propulsion system. This transfer presents a mission risk, because chaotic dynamics in the region around the resonance could cause unplanned trajectory excursions that would negatively impact the mission. This study is motivated by this challenge; not to design Dawn's trajectory but to develop a general transfer design strategy for crossing this potentially hazardous region during future missions.

The chaotic motion associated with resonant orbit dynamics results from a repeating system configuration that allows small

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\*Associate Professor, Departament de Matemàtiques, Edifici C; also Researcher, Institut d'Estudis Espacials de Catalunya; jmm@mat.uab.cat. Member AIAA.

<sup>†</sup>Mission Design Engineer, Jet Propulsion Laboratory, Navigation and Mission Design Section, 4800 Oak Grove Drive, M/S 301-121; Stephen.B .Broschart@jpl.nasa.gov. Senior Member AIAA.

<sup>‡</sup>Assistant Professor, University of California, Irvine, Mechanical and Aerospace Engineering, 4200 Engineering Gateway; bvillac@uci.edu. Member AIAA.