Dynamical Systems
Vol. 25, No. 2, June 2010, 269–284

Global dynamics of a family of 3-D Lotka–Volterra systems

A.C. Murza[a]* and A.E. Teruel[b]

[a]*IFISC, Institut de Física Interdisciplinaria i Sistemes Complexes (CSIC – UIB), Universitat de les Illes Balears, Crta. de Valldemossa km. 7.5, 07122 Palma de Mallorca, Spain;* [b]*Departament de Matemátiques i Informática, Universitat de les Illes Balears, Crta. de Valldemossa km. 7.5, 07122 Palma de Mallorca, Spain*

(*Received 21 July 2009; final version received 21 December 2009*)

In this article, we analyse the flow of a family of three-dimensional Lotka–Volterra systems restricted to an invariant and bounded region. The behaviour of the flow in the interior of this region is simple: either every orbit is a periodic orbit or orbits move from one boundary to another. Nevertheless, the complete study of the limit sets in the boundary allows one to understand the bifurcations which take place in the region as a global bifurcation that we denote by focus-centre-focus bifurcation.

Keywords: Lotka–Volterra system; integrability; first integrals; flow description; limit sets; bifurcation set

AMS Subject Classifications: 34C23; 34C30; 34C15; 37G35

1. Introduction

Consider a closed chemical system composed of four coexisting chemical species denoted by X, Y, Z and V, which represent four possible states of a macromolecule operating in a reaction network far from equilibrium. As discussed by Wyman [1], such a reaction can be modelled as a 'turning wheel' of one-step transitions of the macromolecule, which circulate in a closed reaction path involving the four possible states. The turning wheels have been proposed by Di Cera *et al.* [2] as a generic model for macromolecular autocatalytic interactions.

While Di Cera's model considers unidirectional first-order interactions, Murza *et al.* [3] consider a closed sequence of chemical equilibria. In their approach the reaction rates are defined as functions of the time-dependent product concentrations, multiplied by their reaction rate constants. This type of reaction rates has been introduced in Wyman's original paper [1].

Following the closed sequence of chemical equilibria in [3], the autocatalytic chemical reactions between X, Y, Z and V (Figure 1) are governed by the following 4-parameter family of nonlinear differential equations

$$\begin{aligned}\dot{x} &= x(k_1 y - k_4 v),\\ \dot{y} &= y(k_2 z - k_1 x),\\ \dot{z} &= z(k_3 v - k_2 y),\\ \dot{v} &= v(k_4 x - k_3 z).\end{aligned} \tag{1}$$

*Corresponding author. Email: amurza@ifisc.uib-csic.es

ISSN 1468–9367 print/ISSN 1468–9375 online

DOI: 10.1080/14689360903575196
http://www.informaworld.com