ON NON–SMOOTH PERTURBATIONS OF DEGENERATE OR NON–DEGENERATE PLANAR CENTERS

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ABSTRACT. We provide sufficient conditions for the existence of limit cycles of non–smooth perturbed planar centers, when the set of discontinuity is an algebraic variety. It is introduced a mechanism which allows us to deal with such system, even in higher dimension. The main tool used in this paper is the averaging method. Two applications are given in careful detail.

1. INTRODUCTION

The theory of discontinuous system has been developing at a very fast pace in recent years and it has become certainly an important frontiers between Mathematics, Physics, Engineering and other fields of science. The study of this kind of system is motivated by various applications. For instance, we may cite some problems in control theory [3], nonlinear oscillations [1, 20], non-smooth mechanics [6], economics[12, 16], biology [4], and others.

By the other hand, the knowledge of the existence or not of periodic solutions is very important for understanding the dynamics of the differential systems. One of the useful tools to detect such solutions is the averaging theory, which is a classical and matured tool that provides means to study the behavior of nonlinear smooth dynamical systems. We refer to the book of Sanders and Verhulst [21] and to the book of Verhulst [22] for a general introduction of this subject.

Buica and Llibre [7] generalize the averaging theory for studying periodic solutions of continuous differential systems using mainly the Brouwer degree. More recently, Llibre, Novaes and Teixeira [18], extended the averaging theory for studying periodic solutions of a class of discontinuous piecewise differential system, see Theorem 2. In what follows, we introduce the class of piecewise discontinuous system studied in [18].

Let D be an open subset of \mathbb{R}^n . Let $X, Y : \mathbb{R} \times D \to \mathbb{R}^n$ be two continuous vector fields and $h : \mathbb{R} \times D \to \mathbb{R}$ be a C^1 function. We Assume that the functions

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