



Available online at www.sciencedirect.com



Journal of Differential Equations

J. Differential Equations 259 (2015) 4615-4633

www.elsevier.com/locate/jde

Regularization of hidden dynamics in piecewise smooth flows

Douglas D. Novaes^a, Mike R. Jeffrey^{b,*}

 ^a Departamento de Matemática, Universidade Estadual de Campinas, Rua Sérgio Baruque de Holanda, 651, Cidade Universitária Zeferino Vaz, 13083-859, Campinas, SP, Brazil
^b Department of Engineering Mathematics, University of Bristol, Merchant Venturer's Building, Bristol BS8 1UB, United Kingdom

Received 7 December 2014; revised 2 June 2015

Available online 16 June 2015

Abstract

This paper studies the equivalence between differentiable and non-differentiable dynamics in \mathbb{R}^n . Filippov's theory of discontinuous differential equations allows us to find flow solutions of dynamical systems whose vector fields undergo switches at thresholds in phase space. The canonical *convex combination* at the discontinuity is only the linear part of a *nonlinear combination* that more fully explores Filippov's most general problem: the differential inclusion. Here we show how recent work relating discontinuous systems to singular limits of continuous (or *regularized*) systems extends to nonlinear combinations. We show that if sliding occurs in a discontinuous systems, there exists a differentiable slow–fast system with equivalent slow invariant dynamics. We also show the corresponding result for the *pinching* method, a converse to regularization which approximates a smooth system by a discontinuous one. (© 2015 Elsevier Inc. All rights reserved.

Keywords: Sliding; Discontinuous; Hidden; Regularization; Pinching; Singular perturbation

Corresponding author. *E-mail addresses:* ddnovaes@ime.unicamp.br (D.D. Novaes), mike.jeffrey@bristol.ac.uk (M.R. Jeffrey).

http://dx.doi.org/10.1016/j.jde.2015.06.005 0022-0396/© 2015 Elsevier Inc. All rights reserved.