



A Simple Solution to the Braga–Mello Conjecture

Douglas D. Novaes

*Departament de Matemàtiques, Universitat Autònoma de Barcelona,
08193 Bellaterra, Barcelona, Catalonia, Spain
ddnovaes@mat.uab.cat*

**Departamento de Matemática, Universidade Estadual de Campinas,
Rua Sérgio Barúque de Holanda, 651, Cidade Universitária Zeferino Vaz,
CEP 13083-859, Campinas, São Paulo, Brazil
ddnovaes@ime.unicamp.br*

Enrique Ponce

*Departamento de Matemática Aplicada II,
Escuela Técnica Superior de Ingeniería,
Camino de los Descubrimientos s.n., 41092 Sevilla, Spain
eponcem@us.es*

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Recently Braga and Mello conjectured that for a given $n \in \mathbb{N}$ there is a piecewise linear system with two zones in the plane with exactly n limit cycles. In this paper, we prove a result from which the conjecture is an immediate consequence. Several explicit examples are given where location and stability of limit cycles are provided.

Keywords: Discontinuous piecewise linear differential system; limit cycle; nonsmooth differential system.

1. Introduction and Statement of the Main Results

The computation of upper bounds for the number of limit cycles in all possible configurations within the family of planar piecewise linear differential systems with two zones has been the subject of some recent papers. Assuming that the separation boundary is a straight line, Han and Zhang [2010] conjectured in 2010 that for such planar piecewise linear systems there can be at most two limit cycles. However, Huan and Yang [2012] promptly gave a negative answer to this conjecture by means of a numerical example with three limit cycles under a focus–focus configuration. Such counter-intuitive example led researchers to look for rigorous proofs of this fact,

see [Llibre & Ponce, 2012] for a computer-assisted proof, and [Freire *et al.*, 2014] for an analytical proof under a more general setting.

Recently, in [Braga & Mello, 2013] one can find a study showing that the three limit cycles of Huan and Yang’s example can be simultaneously obtained through a rather special bifurcation. Later, a general and analytical proof for the existence of three nested limit cycles in certain open regions of the parameter space in the focus–focus configuration was given in [Freire *et al.*, 2014]. In [Freire *et al.*, 2013], it is proved that one can have three limit cycles not just in the focus–focus case, as shown in [Freire *et al.*, 2014], so that the lower bounds for the maximum number of limit cycles

*Permanent address of author