J. Math. Anal. Appl. 448 (2017) 171–186

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

On extended Chebyshev systems with positive accuracy

Douglas D. Novaes [a], Joan Torregrosa [b,*]

[a] *Departamento de Matemática, Universidade Estadual de Campinas, Rua Sérgio Buarque de Holanda, 651, Cidade Universitária Zeferino Vaz, 13083-859, Campinas, SP, Brazil*
[b] *Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain*

ARTICLE INFO

Article history:
Received 19 April 2016
Available online 3 November 2016
Submitted by M.J. Schlosser

Keywords:
Number of zeros of real functions
ECT-system
Zeros of Melnikov functions for nonsmooth systems

ABSTRACT

A classical necessary condition for an ordered set of $n + 1$ functions $\mathcal{F}$ to be an ECT-system in a closed interval is that all the Wronskians do not vanish. With this condition all the elements of Span($\mathcal{F}$) have at most n zeros taking into account the multiplicity. Here the problem of bounding the number of zeros of Span($\mathcal{F}$) is considered as well as the effectiveness of the upper bound when some Wronskians vanish. For this case we also study the possible configurations of zeros that can be realized by elements of Span($\mathcal{F}$). An application to count the number of isolated periodic orbits for a family of nonsmooth systems is performed.

1. Introduction and statement of the main results

Let $\mathcal{F} = [u_0, \ldots, u_n]$ be an ordered set of functions of class $\mathcal{C}^r$, $r \geq n$, on the closed interval $[a, b]$. We denote by $Z(\mathcal{F})$ the maximum number of zeros counting multiplicity that any nontrivial function $v \in \text{Span}(\mathcal{F})$ can have. Here Span($\mathcal{F}$) is the set of functions generated by liner combinations of elements of $\mathcal{F}$, that is $v(s) = a_0u_0(s) + a_1u_1(s) + \cdots + a_nu_n(s)$ where a_i, for $i = 0, 1, \ldots, n$, are real numbers.

The theory of Chebyshev systems is a classical tool to study the quantity $Z(\mathcal{F})$. In this theory, when $Z(\mathcal{F}) \leq n$, the set $\mathcal{F}$ is called an *extended Chebyshev system* or ET-system on $[a, b]$, see [7]. When the functions in $\mathcal{F}$ are linearly independent there always exists an element in Span($\mathcal{F}$) with n zeros, see [11]. From this property $Z(\mathcal{F}) = n$ for an ET-system, but in general we can not assure if they are simple or not. This problem will be addressed later. In [6], when $Z(\mathcal{F}) \leq n + k$, the set $\mathcal{F}$ is called an extended Chebyshev system with *accuracy* k on $[a, b]$. From this definition it is natural to consider the lowest possible k. Therefore there exists an element in Span($\mathcal{F}$) with $n + k$ zeros and, consequently, $Z(\mathcal{F}) = n + k$. Indeed, this is the definition used in [4] and it is the one that we shall use throughout the present work. In concrete problems this is not a useful definition in order to get if $\mathcal{F}$ is an ET-system, with accuracy or not.

* Corresponding author.
E-mail addresses: ddnovaes@ime.unicamp.br (D.D. Novaes), torre@mat.uab.cat (J. Torregrosa).

http://dx.doi.org/10.1016/j.jmaa.2016.10.076
0022-247X/