A PERTURBATION OF THE RELATIVISTIC KEPLER PROBLEM

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Abstract. We consider the Kepler Problem with the first order relativistic correction and show that, for a suitable class of perturbations, "almost all" the invariant tori and cylinders of the unperturbed system persist and that the perturbed system has strong evidences of non–integrability.

1. Introduction

Let us consider the first order relativistic correction to the Kepker problem, given by the Hamiltonian

$$\begin{aligned} \bar{H}_\varepsilon : \Re^+ \times S^1 \times \Re^2 &\to \Re \\ (r, \theta, p_r, p_\theta) &\to \frac{p_r^2}{2} + \frac{p_\theta^2 - 2\varepsilon}{2r^2} - \frac{1}{r}, \end{aligned}$$

where (r, θ) are polar coordinates in the plane, p_r and p_θ are the conjugate momenta and ε is a positive real parametre (when $\varepsilon = 0$ this is just the Kepler Hamiltonian). The associated equations of motion are

$$\begin{aligned} \dot{r} &= p_r, \\ \dot{\theta} &= p_\theta r^{-2}, \\ \dot{p}_r &= (p_\theta^2 - 2\varepsilon) r^{-3} - r^{-2}, \\ \dot{p}_\theta &= 0. \end{aligned} \tag{1}$$

* Partially supported by Instituto Nacional de Investigaçao Cientifica.

** Partially supported by a grant of the DGICYT no.BE90-135.

+ Partially supported by a grant of the DGICYT no.PB86-0351.