ROYAL SOCIETY OPEN SCIENCE

royalsocietypublishing.org/journal/rsos

Research

Cite this article: Nurtay A, Hennessy MG, Sardanyés J, Alsedà L, Elena SF. 2019 Theoretical conditions for the coexistence of viral strains with differences in phenotypic traits: a bifurcation analysis. *R. Soc. open sci.* **6**: 181179. http://dx.doi.org/10.1098/rsos.181179

Received: 23 July 2018
Accepted: 26 November 2018

Subject Category:
Mathematics

Subject Areas:
mathematical modelling/health and disease and epidemiology/applied mathematics

Keywords:
bifurcations, epidemiology, infection dynamics, mathematical biology, virus evolution

Authors for correspondence:
Anel Nurtay
e-mail: anurtay@crm.cat
Matthew G. Hennessy
e-mail: mhennessy@crm.cat

THE ROYAL SOCIETY PUBLISHING

Theoretical conditions for the coexistence of viral strains with differences in phenotypic traits: a bifurcation analysis

Anel Nurtay[1,2,3,4], Matthew G. Hennessy[1,2], Josep Sardanyés[1,2], Lluís Alsedà[3,1,2] and Santiago F. Elena[4,5]

[1]Centre de Recerca Matemàtica, [2]Barcelona Graduate School of Mathematics (BGSMath), and [3]Departament de Matemàtiques, Universitat Autònoma de Barcelona, Campus de Bellaterra, Edifici C, 08193 Bellaterra, Spain
[4]Instituto de Biología Integrativa de Sistemas, CSIC-Universitat de València, Parc Científic UV, Paterna, València 46980, Spain
[5]Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, NM 87501, USA

AN, 0000-0001-7107-1656; MGH, 0000-0002-5928-6256; SFE, 0000-0001-8249-5593

We investigate the dynamics of a wild-type viral strain which generates mutant strains differing in phenotypic properties for infectivity, virulence and mutation rates. We study, by means of a mathematical model and bifurcation analysis, conditions under which the wild-type and mutant viruses, which compete for the same host cells, can coexist. The coexistence conditions are formulated in terms of the basic reproductive numbers of the strains, a maximum value of the mutation rate and the virulence of the pathogens. The analysis reveals that parameter space can be divided into five regions, each with distinct dynamics, that are organized around degenerate Bogdanov–Takens and zero-Hopf bifurcations, the latter of which gives rise to a curve of transcritical bifurcations of periodic orbits. These results provide new insights into the conditions by which viral populations may contain multiple coexisting strains in a stable manner.

1. Introduction

The combination of very large population sizes, very short generation times, and lack of proof-reading mechanisms during genome replication confer viral populations with an extremely high evolutionary plasticity that allow them to quickly adapt to

© 2019 The Authors. Published by the Royal Society under the terms of the Creative Commons Attribution License http://creativecommons.org/licenses/by/4.0/, which permits unrestricted use, provided the original author and source are credited.