Geometric and algebraic classification of quadratic differential systems with invariant hyperbolas

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Abstract

Let **QSH** be the whole class of non-degenerate planar quadratic differential systems possessing at least one invariant hyperbola. We classify this family of systems, modulo the action of the group of real affine transformations and time rescaling, according to their geometric properties encoded in the configurations of invariant hyperbolas and invariant straight lines which these systems possess. The classification is given both in terms of algebraic geometric invariants and also in terms of affine invariant polynomials and it yields a total of 205 distinct such configurations. We have 162 configurations for the subclass $\mathbf{QSH}_{(\eta>0)}$ of systems which possess three distinct real singularities at infinity, and 43 configurations for the subclass $\mathbf{QSH}_{(\eta=0)}$ of systems which possess either exactly two distinct real singularities at infinity or the line at infinity filled up with singularities. The algebraic classification, based on the invariant polynomials, is also an algorithm which makes it possible to verify for any given real quadratic differential system if it has invariant hyperbolas or not and to specify its configuration of invariant hyperbolas and straight lines.

Key-words: quadratic differential systems, algebraic solution, configuration of algebraic solutions, affine invariant polynomials, group action

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1 Introduction and statement of the main results

We consider planar polynomial differential systems which are systems of the form

$$dx/dt = p(x,y), \quad dy/dt = q(x,y) \tag{1}$$

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