Family of quadratic differential systems with invariant hyperbolas: a complete classification in the space $\mathbb{R}^{12}$

Regilene D. S. Oliveira
Instituto de Ciências Matemáticas e de Computação
Universidade de São Paulo (regilene@icmc.usp.br)

Alex C. Rezende
Instituto de Ciências Matemáticas e de Computação
Universidade de São Paulo (alexcrezende@gmail.com)

Nicolae Vulpe
Institute of Mathematics and Computer Science
Academy of Sciences of Moldova (nvulpe@gmail.com)

Abstract

In this article we consider the class **QS** of all non-degenerate quadratic systems. A quadratic polynomial differential system can be identified with a single point of $\mathbb{R}^{12}$ through its coefficients. In this paper using the algebraic invariant theory we provided necessary and sufficient conditions for a system in **QS** to have at least one invariant hyperbola in terms of its coefficients. We also considered the number and multiplicity of such hyperbolas. We give here the global bifurcation diagram of the class **QS** of systems with invariant hyperbolas. The bifurcation diagram is done in the 12–dimensional space of parameters and it is expressed in terms of polynomial invariants. The results can therefore be applied for any family of quadratic systems in this class, given in any normal form.

Key-words: quadratic differential systems, invariant hyperbola, affine invariant polynomials, group action

2000 Mathematics Subject Classification: 34C23, 34A34

1 Introduction and statement of main results

We consider here differential systems of the form

$$\frac{dx}{dt} = P(x, y), \qquad \frac{dy}{dt} = Q(x, y), \tag{1}$$

where P, $Q \in \mathbb{R}[x, y]$, i.e. P, Q are polynomials in x, y over $\mathbb{R}$ and their associated vector fields

$$X = P(x, y)\frac{\partial}{\partial x} + Q(x, y)\frac{\partial}{\partial y}. \tag{2}$$