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Research paper

Analytical and numerical results on families of n-ejection-collision orbits in the RTBP

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ABSTRACT

In the planar RTBP with mass ratio μ we regularise the singularity at one of the primaries by means of Levi-Civita's transformation in a rotating frame. We solve the variational equations in a neighbourhood of the ejection/collision orbits, giving analytic expressions for the first terms in μ of the convergent expansion for orbits with eccentricity $e \simeq 1$. For high enough values of the Jacobi constant C we give analytic expressions for the coefficients of the above expansion in powers of the small parameter $1/\sqrt{C}$ and we prove the existence of four families of the so called n-ejection-collision (EC) orbits, that are orbits which eject from the primary and reach n relative maxima in the distance with the primary before finally colliding with it. Moreover, massive numerical explorations extending the analytical result for any value of the mass ratio and bigger ranges of C are also shown and discussed.

1. Introduction

We recall that the planar restricted three-body problem (PRTBP) is the problem of the motion of a massless particle in the gravitational field created by two bodies, the *primaries*, of mass μ (the *mass ratio*) and $1-\mu$, which move around circular orbits around their centre of mass. Although this problem has been studied by many different authors, the so called ejection-collision orbits remain to be well understood.

In this paper we state and prove a theorem on the existence of only four families of n-ejection-collision orbits, for any $n \geq 1$, μ small enough and sufficiently large values of the Jacobi constant C. The argument that guarantees the existence of exactly four families relies on the application of a perturbative approach and the implicit function theorem as well. These families will be labelled by α_n, β_n, δ_n and γ_n. The case $n=1$ was already proved in [5] using McGehee's regularisation, which essentially consists of blowing up the singularity to an invariant manifold with two submanifolds of unstable equilibrium points which have stable and unstable manifolds. Ejection-collision orbits are then seen as heteroclinic connections between equilibrium points representing respectively the ejection and the collision. For $\mu = 0$, every ejection orbit is a collision orbit after reaching its apocentre. For $\mu \neq 0$ but small it was seen in [5] that only four of them survive. This approach, however, seems difficult to generalise to a class of orbits which eject from the primary, have $n \geq 1$ apocentres with $n-1$ close approaches to the primary with no collision and finally collide with it (the n-EC orbits). The difficulty is due to the fact that it is not easy to follow, even numerically, an orbit which gets several times very close to a hyperbolic equilibrium point as the interval of time becomes unbounded. The way out has been to use Levi-Civita's regularisation, which transforms the

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