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On a class of invariant algebraic curves for Kukles systems

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Abstract. In this paper we give a new upper bound for the degree of a class of transversal to infinity invariant algebraic curves for polynomial Kukles systems of arbitrary degree. Moreover, we prove that a quadratic Kukles system having at least one transversal to infinity invariant algebraic curve is integrable.

Keywords: Kukles system, invariant curve, integrability, limit cycle.

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1 Introduction

Darboux in 1878 published his seminal works [4] and [5], where he showed that a planar polynomial differential system with a sufficient number of invariant algebraic curves has a first integral. Since that time, the research and computation of invariant curves in planar polynomial vector fields has been intensive. See [2,3,6–8] and references there in. However, to determine whether a concrete planar polynomial system has invariant algebraic curves or not, as well as the properties of such curves: degree, connected components, etc. can be extremely difficult problems.

In this work, we consider real Kukles systems of the form

$$\dot{x} = -y, \qquad \dot{y} = Q(x,y), \tag{1.1}$$

where $Q(x,y)$ is a real polynomial of degree at least two and without y as a divisor.

Our main result is the following.

Theorem 1.1. *Let $\{F = 0\}$ be a transversal to infinity invariant algebraic curve of degree n of the Kukles system (1.1) of degree $d \geq 3$. Supposes that x is not a divisor of the higher degree homogeneous part of F. Then*

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