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Existence and uniqueness of limit cycles for generalized φ-Laplacian Liénard equations

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ABSTRACT

The Liénard equation $x'' + f(x)x' + g(x) = 0$ appears as a model in many problems of science and engineering. Since the first half of the 20th century, many papers have appeared providing existence and uniqueness conditions for limit cycles of Liénard equations. In this paper we extend some of these results for the case of the generalized φ-Laplacian Liénard equation, $(\varphi(x'))' + f(x)\psi(x') + g(x) = 0$. This generalization appears when derivations of the equation different from the classical one are considered. In particular, the relativistic van der Pol equation, $\left(x'/\sqrt{1-(x'/c)^2}\right)' + \mu(x^2-1)x' + x = 0$, has a unique periodic orbit when $\mu \neq 0$.

1. Introduction

The Liénard equation,

$$x'' + f(x)x' + g(x) = 0, \tag{1}$$

appears as a model in many areas in science and engineering. It was intensively studied during the first half of the 20th century as it can be used to model oscillating circuits or simple pendulums. In the case of the simple pendulum, the functions f and g represent the friction and acceleration terms. One of the first models where this equation appeared was introduced by van der Pol [12], considering the equation

$$x'' + \mu(x^2 - 1)x' + x = 0,$$

for modeling the oscillations of a triode vacuum tube. See [7] for other references about more applications.

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