# HOMOGENEOUS POLYNOMIAL VECTOR FIELDS ON $\mathbb{S}^2$

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**Abstract.** The theory of dynamical systems is one of the most important tools for providing qualitative and quantitative models of the applied sciences. Since the first works published by Poincaré, the qualitative theory of ordinary differential equations has suffered a significant expansion involving technics of almost all areas of mathematics. Inside this theory the vector fields defined on the plane or on a surface have been one of the main studied objects. However, these topics are far from being totaly understood. Famous open problems in this subject are the $16^{th}$ Hilbert's problem, the Center–Focus problem, the Integrability problem, etc. Recently new knowledge about the Darboux theory of integrability and about invariant algebraic curves provide important contributions to some of these problems.

In this work we consider homogeneous polynomial vector fields on the 2–dimensional sphere $\mathbb{S}^2$. We study their invariant circles, i.e. invariant algebraic curves on $\mathbb{S}^2$ under the flow associated to such vector fields formed by circles. We determine upper bounds for the maximum number of invariant circles of a homogeneous polynomial vector field on $\mathbb{S}^2$ in function of its degree, when this number is finite. Moreover, we provide almost a global classification of the phase portraits of the homogeneous polynomial vector fields on $\mathbb{S}^2$ of degree 2. For doing that the main tool that we use is the qualitative theory of the vector fields in the plane, because the homogeneous polynomial vector fields on $\mathbb{S}^2$ of degree 2 can be reduced to study a family of planar polynomial vector fields of degree 3 with six parameters.

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