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CANARD TRAJECTORIES IN 3D PIECEWISE LINEAR SYSTEMS

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Abstract. We present some results on singularly perturbed piecewise linear systems, similar to those obtained by the Geometric Singular Perturbation Theory. Unlike the differentiable case, in the piecewise linear case we obtain the global expression of the slow manifold $\mathcal{S}_\varepsilon$. As a result, we characterize the existence of canard orbits in such systems. Finally, we apply the above theory to a specific case where we show numerical evidences of the existence of a canard cycle.

1. **Introduction and main results.** Singularly perturbed systems of ordinary differential equations are written in standard form as

$$\dot{\mathbf{u}} = \frac{d\mathbf{u}}{d\tau} = g(\mathbf{u}, \mathbf{v}, \varepsilon), \quad \varepsilon \dot{\mathbf{v}} = \varepsilon \frac{d\mathbf{v}}{d\tau} = f(\mathbf{u}, \mathbf{v}, \varepsilon), \tag{1}$$

where $(\mathbf{u}, \mathbf{v}) \in \mathbb{R}^s \times \mathbb{R}^q$ are the state variables, f and g are sufficiently smooth functions and $0 < \varepsilon \ll 1$ is a small parameter. From the expression above, the coordinates of $\mathbf{u}$ are called slow variables, while the coordinates of $\mathbf{v}$ are called fast variables. The variable τ is referred to as the slow time. Changing the time τ to the fast time $t = \tau/\varepsilon$, system (1) is written as

$$\mathbf{u}' = \frac{d\mathbf{u}}{dt} = \varepsilon g(\mathbf{u}, \mathbf{v}, \varepsilon), \quad \mathbf{v}' = \frac{d\mathbf{v}}{dt} = f(\mathbf{u}, \mathbf{v}, \varepsilon). \tag{2}$$

Systems (1) and (2) are differentiably equivalent and their phase portraits are the same. It can be understood that the dynamics of both systems exhibit an slow-fast explicit splitting. In this setting, system (1) and (2) are called slow-fast systems. Usually, system (1) is referred to as the slow system whereas system (2) is called the fast system.

Fenichel's geometric theory [13] allows us to analyse the dynamics of the perturbed system (1) by combining the behaviour of the singular orbits, corresponding to the limiting cases given by $\varepsilon = 0$. In particular, by setting $\varepsilon = 0$ in (1) and in (2), we get respectively the reduced problem

$$\dot{\mathbf{u}} = g(\mathbf{u}, \mathbf{v}, 0), \quad \mathbf{0} = f(\mathbf{u}, \mathbf{v}, 0), \tag{3}$$

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