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Shape and period of limit cycles bifurcating from a class of Hamiltonian period annulus

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ABSTRACT

In this work we are concerned with the problem of shape and period of isolated periodic solutions of perturbed analytic radial Hamiltonian vector fields in the plane. Françoise developed a method to obtain the first non vanishing Poincaré–Pontryagin–Melnikov function. We generalize this technique and we apply it to know, up to any order, the shape of the limit cycles bifurcating from the period annulus of the class of radial Hamiltonians. We write any solution, in polar coordinates, as a power series expansion in terms of the small parameter. This expansion is also used to give the period of the bifurcated periodic solutions. We present the concrete expression of the solutions up to third order of perturbation of Hamiltonians of the form $H = H(r)$. Necessary and sufficient conditions that show if a solution is simple or double are also presented.

1. Introduction

Consider a Hamiltonian vector field having a continuous domain of closed trajectories (period annulus) in a neighborhood of an equilibrium point. A classical problem in perturbation theory is the boundedness of the number of isolated periodic solutions bifurcating from the period annulus through analytic perturbations. In the past years the shape and period of such periodic solutions have also been studied. Let us consider the equation

$$\begin{cases} \dot{x} = -\dfrac{\partial}{\partial y} H(x, y) + \varepsilon P(x, y, \varepsilon), \\ \dot{y} = \dfrac{\partial}{\partial x} H(x, y) + \varepsilon Q(x, y, \varepsilon), \end{cases} \tag{1}$$

where $H(x, y)$, $P(x, y, \varepsilon)$ and $Q(x, y, \varepsilon)$ are analytic functions and ε is a small parameter. Let us assume that, in (r, θ)-polar coordinates, H only depends on r, $H = H(r)$, and that the equilibrium point is at the origin. When $\varepsilon = 0$, we call this equation of Hamiltonian radial type. In this work we are involved with former problems concerning the perturbed Hamiltonian equation (1).

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