CORRIGENDUM TO "SHAPE AND PERIOD OF LIMIT CYCLES BIFURCATING FROM A CLASS OF HAMILTONIAN PERIOD ANNULUS"[NONLINEAR ANAL. 81 (2013) 130–148]

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Abstract. In our paper [1] we are concerned with the problem of shape and period of isolated periodic solutions of perturbed analytic radial Hamiltonian vector fields in the plane. Actually, there is a mistake in the formula of the first order approximation of the period given in Corollary 4. Here we give its proper drafting.

Corollary 4 of [1] provides the expression for the period function of the limit cycles bifurcating from the period annulus of the class of radial Hamiltonian differential equations in the plane given by

$$\left\{ \begin{array}{rcl} \dot{x} & = & -\frac{\partial}{\partial y} H(x,y) + \varepsilon P(x,y,\varepsilon), \\ \dot{y} & = & \frac{\partial}{\partial x} H(x,y) + \varepsilon Q(x,y,\varepsilon), \end{array} \right. \tag{1}$$

where $H(x,y)$, $P(x,y,\varepsilon)$ and $Q(x,y,\varepsilon)$ are analytic functions and ε is a small parameter. We assume that this Hamiltonian vector field has a continuum of periodic orbits around the origin. In (r,θ)-polar coordinates, H only depends on r, $H = H(r)$ and the differential system (1) is written as the one-form

$$dH + \sum_{i=1}^{\infty} \varepsilon^i \left(S_i(r,\theta)\, dr - R_i(r,\theta)\, d\theta \right) = 0. \tag{2}$$

The general expression for the period of the isolated periodic orbits of (2) given in [1] is correct but there is a mistake in the first order term in its series in ε. The corrected expression is done in the corollary below. We remark that only the general expression for the period is used in [1]. Hence, all the expressions for the period described in the applications are correct.

Corollary 4. *Let us assume the hypotheses of Theorem 1. Then, the period of the periodic solution, $r(\theta;\rho,\varepsilon)$, of equation (2) given in Theorem 3 satisfies*

$$T(\varepsilon;\rho) = \int_0^{2\pi} \frac{r(\theta;\rho,\varepsilon)}{H'(r(\theta;\rho,\varepsilon)) + \sum_{i=1}^{\infty} \varepsilon^i S_i(r(\theta;\rho,\varepsilon),\theta)} d\theta. \tag{3}$$

In particular,

$$T(\varepsilon;\rho) = \frac{2\pi\rho}{H'(\rho)} + \varepsilon \left(\frac{2\pi F_2(\rho)\,(\rho H''(\rho) - H'(\rho))}{(H'(\rho))^2 F_1'(\rho)} - \frac{\rho}{(H'(\rho))^2} \int_0^{2\pi} S_1(\rho,\theta)\, d\theta \right) + O(\varepsilon^2). \tag{4}$$

Proof. Assume that, in equation (1),

$$P(x,y,\varepsilon) = \sum_{i=0}^{\infty} \varepsilon^i P_i(x,y), \quad Q(x,y,\varepsilon) = \sum_{i=0}^{\infty} \varepsilon^i Q_i(x,y),$$

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where P_i, Q_i are analytic functions. Hence, in polar coordinates $(x, y) = (r\cos\theta, r\sin\theta)$, the angle variation of equation (1) writes as

$$\frac{d\theta}{dt} = \frac{1}{r}\frac{\partial H}{\partial r}(r,\theta) + \frac{1}{r}\sum_{i=1}^{\infty} \varepsilon^i S_i(r,\theta), \tag{5}$$

where

$$S_i(r,\theta) = \cos\theta\, Q_i(r\cos\theta, r\sin\theta) - \sin\theta\, P_i(r\cos\theta, r\sin\theta).$$

Since equation (1) is of radial Hamiltonian type we write $H'(r) = \frac{\partial H}{\partial r}(r,\theta)$ and, hence, (5) writes as the 1-form

$$dt = \frac{r\, d\theta}{H'(r) + \sum\limits_{i=1}^{\infty} \varepsilon^i S_i(r,\theta)}. \tag{6}$$

Expression (3) follows from (6) by direct integration on t and by taking into account that $r = r(\theta; \rho, \varepsilon)$.

To obtain formula (4), first we develop the integrand of expression (3), up to first order, in ε-power series and we get

$$T(\varepsilon;\rho) = \frac{2\pi\rho}{H_r'(\rho)} - \frac{\varepsilon}{(H_r'(\rho))^2}\int_0^{2\pi} \big((\rho H''(\rho) - H'(\rho))\, r_1(\theta) + \rho S_1(\rho,\theta) \big)\, d\theta, \tag{7}$$

where $r_1(\theta)$ is given by the development of $r(\theta;\rho,\varepsilon)$ in its ε-power series

$$r(\theta;\rho,\varepsilon) = r_0(\theta) + \varepsilon r_1(\theta) + \varepsilon^2 r_2(\theta) + \varepsilon^3 r_3(\theta) + \cdots .$$

Finally, by using the expression of $r_1(\theta)$, given in Theorem 1, and Theorem 3.(i) of [1], formula (4) follows.

□

References

[1] Prohens, R. and Torregrosa, J. Shape and period of limit cycles bifurcating from a class of Hamiltonian period annulus. *Nonlinear Anal. Ser. A*, **81** (2013) 1307–148.

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