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Periodic orbits from second order perturbation via rational trigonometric integrals

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HIGHLIGHTS

- Along the number, we study the shape and the period of the perturbed periodic orbits.
- First a second order studies are given for concrete planar vector fields.
- Same technique apply simultaneously to Abel equations and planar vector fields.
- The computations share the same integrals of rational trigonometric functions.
- Isochronous quadratic systems have at most 2 limit cycles up to second order.

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ABSTRACT

The second order Poincaré-Pontryagin-Melnikov perturbation theory is used in this paper to study the number of bifurcated periodic orbits from certain centers. This approach also allows us to give the shape and the period up to the first order. We address these problems for some classes of Abel differential equations and quadratic isochronous vector fields in the plane. We prove that two is the maximum number of hyperbolic periodic orbits bifurcating from the isochronous quadratic centers with a birational linearization under quadratic perturbations of second order. In particular the configurations (2, 0) and (1, 1) are realizable when two centers are perturbed simultaneously. The required computations show that all the considered families share the same iterated rational trigonometric integrals.

1. Introduction

In this paper we study the number, the shape and the period of closed trajectories bifurcating from the period annuli for some families of differential equations. We focus our attention on the second order analysis of the perturbed equation

$$\frac{dr}{d\theta} = a_0(r,\theta) + \varepsilon\, a_1(r,\theta) + \varepsilon^2\, a_2(r,\theta) + O(\varepsilon^3). \tag{1}$$

Here a_i is an analytic function, 2π-periodic in $\theta \in [-\pi, \pi]$, for $i = 0, 1, 2$. We denote by $r(\theta; \rho, \varepsilon)$ the solution of (1) satisfying $r(-\pi; \rho, 0) = \rho$. We will assume throughout this paper that ε is small enough. The power series in ε of this solution is written as

$$r(\theta; \rho, \varepsilon) = r_0(\theta; \rho) + \varepsilon\, r_1(\theta; \rho) + \varepsilon^2\, r_2(\theta; \rho) + O(\varepsilon^3). \tag{2}$$

We refer to this expansion as the *shape of the orbit*. Additionally we assume that, when $\varepsilon = 0$, Eq. (1) has a *period annulus*; that is, an open continuum neighborhood of periodic solutions. In particular, there exists an open interval where the function $r_0(\theta; \rho)$ is 2π-periodic in $\theta \in [-\pi, \pi]$, for every ρ in this interval.

In the concrete case $a_0(r, \theta) = a(\theta)r^2$ we have $r_0(\theta; \rho) = \rho/(1 - \rho A(\theta))$ where $A(\theta) = \int_{-\pi}^{\theta} a(\psi)\, d\psi$. Note that A is also an analytic 2π-periodic function. Furthermore, in a first order analysis of the first return map associated to the period annulus, the

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