Medium amplitude limit cycles of some classes of generalized Liénard systems

Salomón Rebollo-Perdomo

Centre de Recerca Matemàtica,
08193 Bellaterra, Barcelona, Spain.
srebollo@crm.cat

1. Introduction and statement of the results

The bifurcation of limit cycles by perturbing a planar system which has a continuous family of *cycles*, i.e. periodic orbits, has been an intensively studied phenomenon; see for instance [13, 16, 2] and references therein. The simplest planar system having a continuous family of cycles is the linear center, and a special family of its perturbations is given by the generalized polynomial Liénard systems:

$$\dot{x} = y + \sum_{i=1}^{\mu} \varepsilon^i F_i(x), \quad \dot{y} = -x + \sum_{i=1}^{\nu} \varepsilon^i g_i(x), \qquad (1_\varepsilon)$$

where $\mu, \nu \in \mathbb{N}$, $g_i(x)$ and $F_i(x)$ are polynomials for $i \geq 1$, and ε is a small parameter.

The classical and generalized Liénard systems appear very often in several branches of science and engineering, as biology, chemistry, mechanics, electronics, etc., see for instance [20] and references therein. In particular Liénard systems are frequent specially in physiological processes, see for instance [10]. Further, some planar systems can be transformed into (generalized) Liénard systems, see for example [5, 15]. In addition, the generalized polynomial Liénard systems is one of the most considered families in the study of limit cycles, see [18].

We assume $F_\mu(x) \not\equiv 0$ and $g_\nu(x) \not\equiv 0$, then we define

$$m = \max_{1 \leq i \leq \mu} \{\deg F_i(x)\}$$

and

$$n = \max_{1 \leq i \leq \nu} \{\deg g_i(x)\}.$$

For a small enough ε, let $\mathcal{H}_\nu^\mu(m,n)$ be the maximum number of limit cycles of (1_ε) that bifurcate from cycles of the *linear center* (1_0), i.e. the maximum number of *medium amplitude limit cycles* which can bifurcate from (1_0) under the perturbation (1_ε), in short

$$\mathcal{H}_\nu^\mu(m,n) = \left\{ \begin{array}{c} \text{Maximum number of medium} \\ \text{amplitude limit cycles of } (1_\varepsilon) \end{array} \right\}.$$

The main problem concerning $\mathcal{H}_\nu^\mu(m,n)$ is finding its exact value.

Preprint submitted to Elsevier *September 9, 2014*