Qual. Theory Dyn. Syst.
DOI 10.1007/s12346-015-0185-5

Qualitative Theory
of Dynamical Systems

Poincaré–Pontryagin–Melnikov Functions for a Type of Perturbed Degenerate Hamiltonian Equations

Salomón Rebollo-Perdomo[1]

Received: 20 October 2015 / Accepted: 14 December 2015

Abstract In this paper we consider polynomial perturbations of a family of polynomial Hamiltonian equations whose associated Hamiltonian is not transversal to infinity, and its complexification is not a Morse polynomial. We look for an algorithm to compute the first non-vanishing Poincaré–Pontryagin–Melnikov function of the displacement function associated with the perturbed equation. We show that the algorithm of the case when the Hamiltonian is transversal to infinity and its complexification is a Morse polynomial can be extended to our family of perturbed equations. We apply the result to study the maximum number of zeros of the first non-vanishing Poincaré–Pontryagin–Melnikov function associated with some perturbed Hamiltonian equations.

Keywords Abelian integral · Limit cycle · Hamiltonian equation

Mathematics Subject Classification Primary 34C07; Secondary 34C08 · 37G15

1 Introduction

Consider the *perturbed Hamiltonian differential equation*

$$dF - \varepsilon\omega(\varepsilon) = 0, \tag{1_ε}$$

where $F = F(x, y)$ is a real polynomial, and $\omega(\varepsilon) = A(x, y, \varepsilon)dx + B(x, y, \varepsilon)dy$ is a 1-form on the real plane $\mathbb{R}^2$ such that $A(x, y, \varepsilon)$ and $B(x, y, \varepsilon)$ are real poly-

✉ Salomón Rebollo-Perdomo
srebollo@ubiobio.cl

[1] Departamento de Matemática, Universidad del Bío-Bío, Avda. Collao 1202, Concepción, Chile

Published online: 07 January 2016