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Global phase portraits of a SIS model

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ABSTRACT

In the qualitative theory of ordinary differential equations, we can find many papers whose objective is the classification of all the possible topological phase portraits of a given family of differential system. Most of the studies rely on systems with real parameters and the study consists of outlining their phase portraits by finding out some conditions on the parameters. Here, we studied a susceptible-infected-susceptible (SIS) model described by the differential system $\dot{x} = -bxy - mx + cy + mk$, $\dot{y} = bxy - (m + c)y$, where b, c, k, m are real parameters with $b \neq 0$, $m \neq 0$ [2]. Such system describes an infectious disease from which infected people recover with immunity against reinfection. The integrability of such system has already been studied by Nucci and Leach (2004) [7] and Llibre and Valls (2008) [5]. We found out two different topological classes of phase portraits.

1. Introduction

There is a long time that scientists have been curious about the interaction between portions of a population with particular characteristics, e.g. prey and predator interaction [1]. In 1838, Verhulst [13] proposed the study of the population growth by means of the so called logistic equation (see also [1]). In contrast, this equation has had other applications, for instance in the study of the spread and the evolution of diseases. The means of how diseases spread has stimulated the interest mainly within researchers of the biological field [9].

As suggested above, the application of mathematical tools in different areas of sciences has been frequent and has had a great impact on the scientific and/or experimental conclusions. For example, Lotka (1925) and Volterra (1926) described independently the dynamics present in biological interactions between two species using ordinary differential equations (ODE). Their model is now known as the Lotka–Volterra equations and they are the most studied equations in the qualitative theory of ODE (see [1,8] for more details).

Another approach of the ODE is their application in the study of infectious diseases. According to Levin [4], this study represents one of the oldest and richest areas in mathematical biology. Besides, continuous models composed by ODE have formed a large part of the traditional mathematical epidemiology literature, mainly because mathematicians have been attracted by applying the ODE's tools, such as their qualitative theory, to the study of infectious diseases in the attempt of using mathematics to contribute positively to the science field and because the mathematical models become indispensable to inform decision-making.

Particularly, we consider the system of first-order ODE

$$\begin{aligned} \dot{x} &= -bxy - mx + cy + mk, \\ \dot{y} &= bxy - (m + c)y, \end{aligned} \tag{1}$$

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