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Asymptotic Expansion of the Heteroclinic Bifurcation for the Planar Normal Form of the 1:2 Resonance

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We consider the family of planar differential systems depending on two real parameters

$$\dot{x} = y, \quad \dot{y} = \delta_1 x + \delta_2 y + x^3 - x^2 y.$$

This system corresponds to the normal form for the 1:2 resonance which exhibits a heteroclinic connection. The phase portrait of the system has a limit cycle which disappears in the heteroclinic connection for the parameter values on the curve $\delta_2 = c(\delta_1) = -\frac{1}{5}\delta_1 + O(\delta_1^2)$, $\delta_1 < 0$. We significantly improve the knowledge of this curve in a neighborhood of the origin.

Keywords: Homoclinic connections; planar systems; bifurcation diagram.

1. Introduction

The theory of bifurcations is concerned to describe the variation of the qualitative behavior of the phase portrait when we vary the parameters. The bifurcation diagram consists of a partition of the parameter space such that, for parameters belonging to the same region, the respective phase portraits are topologically equivalent.

Consider a two-parameter family of differential systems in the plane. Suppose that there exists a curve such that for parameter values on this curve the systems exhibit heteroclinic connections. Moreover, suppose that when the parameters cross this curve we have the birth of an isolated periodic orbit, that is a limit cycle. In general, the existence of these curves is obtained by topological methods and very few properties are known about its asymptotic development.

For each $\sigma = \pm 1$, the differential system

$$(\dot{x}, \dot{y}) = (y, \delta_1 x + \delta_2 y + \sigma x^3 - x^2 y)$$

is a versal deformation of a system with a double zero eigenvalue with a symmetry of order 2 ($\delta_1 = \delta_2 = 0$). These problems of codimension two were studied by Carr and Horozov in [Carr, 1981; Horozov, 1979]. The unperturbed system is invariant under a rotation of the plane through an angle $2\pi/q$ for $q = 2$ and it can be proved that these are the normal forms for the 1:2 resonance, see [Chow *et al.*, 1994; Kuznetsov, 1998]. The bifurcation diagram of this family is well known and a detailed analysis of them can be obtained in [Kuznetsov, 1998, Section 9.5.3] or [Chow *et al.*, 1994, Section 4.2]. A heteroclinic connection appears for $\sigma = +1$. For $\sigma = -1$ a double homoclinic connection and a double limit cycle bifurcation occur. In particular, the two limit cycles that emerge from the origin from a