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Qualitative Theory of Dynamical Systems

The Monotonicity of the Apsidal Angle Using the Theory of Potential Oscillators

D. Rojas[1]

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Abstract In a central force system the angle between two successive passages of a body through pericenters is called the apsidal angle. In this paper we prove that for central forces of the form $f(r) \sim \lambda r^{-(\alpha+1)}$ with $\alpha < 2$ the apsidal angle is a monotonous function of the energy, or equivalently of the orbital eccentricity.

Keywords Central force · Apsidal angle · Monotonicity · Period function · Two-body problem

Mathematics Subject Classification 70F05 · 70F15 · 34C25

1 Introduction

The angle between two successive passages of a body in a central force system through pericenters is called the apsidal angle Θ. Its behaviour has attracted both physicists and mathematicians attention since 1687 when Newton stated its precession theorem (Book I, Philosophiæ Naturalis Principia Mathematica): for orbits close to the circular ones, a force proportional to $1/r^{\alpha+1}$ leads to $\Theta = \pi/\sqrt{2-\alpha}$. An important and immediate consequence of this result is that experimental measurements of the apsidal angle close to central orbits may give the exponent of the force law. In 1873 Bertrand [1] published a note in Comptes Rendus to prove that among all central field of forces in the Euclidean space there are only two exceptional cases (the harmonic oscillator and the Newtionian potential) in which all solutions close to the circular motions are also periodic. The equivalence of Bertrand's theorem in terms of the apsidal angle is

✉ D. Rojas
rojas@ugr.es

1 Departamento de Matemática Aplicada, Universidad de Granada, Granada, Spain