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BIFURCATION OF RELATIVE EQUILIBRIA GENERATED BY A CIRCULAR VORTEX PATH IN A CIRCULAR DOMAIN

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Abstract. We study the passive particle transport generated by a circular vortex path in a 2D ideal flow confined in a circular domain. Taking the strength and angular velocity of the vortex path as main parameters, the bifurcation scheme of relative equilibria is identified. For a perturbed path, an infinite number of orbits around the centers are persistent, giving rise to periodic solutions with zero winding number.

1. **Introduction.** The passive particle transport in a 2D incompressible inviscid flow with prescribed vorticity is a research topic of the highest relevance in Fluid Dynamics [2, 8, 10]. In the Lagrangian formulation, the advection of single particles is ruled by a Hamiltonian system where the stream function plays the role of the Hamiltonian. In this paper, we consider the dynamics induced in an ideal flow confined in a circular domain of radius R under the action of a prescribed T-periodic vortex path. Such dynamics model the stirring process of an agitator plunged into a fluid inside a cylindrical tank free surface. The main interest of this model is to investigate the amount of fluid that actually mix and how the path of the vortex affects stirring. We refer to [1, 3, 4, 6] and references therein for more information on the model and its historical overview in the literature.

Let $B_R \subset \mathbb{R}^2$ be the open ball of center $(0,0)$ and radius R, and consider a T-periodic vortex path given by $z : \mathbb{R} \to B_R$. Then, the stream function of the fluid confined in B_R and under the action of the vortex is given by

$$\Psi(t,\zeta) = \frac{\Gamma}{2\pi}\left(\ln|\zeta - z(t)| - \ln\left|\zeta - \frac{R^2}{|z(t)|^2}z(t)\right|\right).$$

Here, Γ is the strength or charge of the vortex, and its sign gives the sense of rotation. In this function, the first term accounts for the vortex action, whereas the

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