## A criticality result for polycycles in a family of quadratic reversible centers

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Abstract. We consider the family of dehomogenized Loud's centers  $X_{\mu} = y(x-1)\partial_x + (x+Dx^2+Fy^2)\partial_y$ , where  $\mu = (D, F) \in \mathbb{R}^2$ , and we study the number of critical periodic orbits that emerge or dissapear from the polycycle at the boundary of the period annulus. This number is defined exactly the same way as the well-known notion of cyclicity of a limit periodic set and we call it criticality. The previous results on the issue for the family  $\{X_{\mu}, \mu \in \mathbb{R}^2\}$  distinguish between parameters with criticality equal to zero (regular parameters) and those with criticality greater than zero (bifurcation parameters). A challenging problem not tackled so far is the computation of the criticality of the bifurcation parameters, which form a set  $\Gamma_B$  of codimension 1 in  $\mathbb{R}^2$ . In the present paper we succeed in proving that a subset of  $\Gamma_B$  has criticality equal to one.

## **1** Introduction and statement of the result

In the present paper we study the local bifurcation diagram of the period function associated to a family of quadratic centers. By local we mean near the polycycle at the boundary of the period annulus of the center. In the literature one can find different terminology to classify the quadratic centers but essentially there are four families: Hamiltonian, reversible  $Q_3^R$ , codimension four  $Q_4$  and generalized Lotka-Volterra systems  $Q_3^{LV}$ . According to Chicone's conjecture [2], the reversible centers have at most two critical periodic orbits, whereas the centers of the other three families have monotonic period function. In this context critical periodic orbits play exactly the same role as limit cycles in the celebrated Hilbert's 16th problem (see for instance [6] and references therein). What is more, from the point of view of the techniques, results and notions involved, Chicone's conjecture is the counterpart for the period function to the question of whether quadratic polynomial differential systems have at most four limit cycles, i.e., H(2) = 4. Both problems are far from being solved and pose challenging difficulties. There are many papers proving partial results related to Chicone's conjecture and there is much analytic evidence that it is true. In this direction, and without being exhaustive, let us quote Coppel and Gavrilov [4], who showed that the period function of any Hamiltonian quadratic center is monotonic, and Zhao [18], who proved the same property for the  $Q_4$  and yean proved in [4], solved independently the monotonicity of the classical Lotka-Volterra centers, which constitute a hypersurface inside the  $Q_3^{LV}$  family, and more recently the same property has been proved in [16] for two other hypersurfaces. With regard to reversible quadratic centers, it is well known that they can be brought by an affine transformation and a constant rescaling of time to the *Loud normal form* 

$$\begin{cases} \dot{x} = -y + Bxy, \\ \dot{y} = x + Dx^2 + Fy^2. \end{cases}$$

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