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## Normal forms for rational difference equations with applications to the global periodicity problem \*

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## Abstract

We propose a classification and derive the associated normal forms for rational difference equations with complex coefficients. As an application, we study the global periodicity problem for second-order rational difference equations with complex coefficients. We find new necessary conditions as well as some new examples of globally periodic equations.

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## 1. Introduction

Rational difference equations (linear fractional) have been proposed as a paradigmatic family of equations in order to study the dynamical issues of discrete systems in  $\mathbb{R}^n$  or  $\mathbb{C}^n$ , [18]. In this paper we deal with equations of the type

$$x_{n+k} = \frac{a_0 + a_1 x_n + a_2 x_{n+1} + \dots + a_k x_{n+k-1}}{b_0 + b_1 x_n + b_2 x_{n+1} + \dots + b_k x_{n+k-1}},$$
(1)

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