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On the enveloping method and the existence of global Lyapunov functions

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An interpretation of Cull's enveloping method used to determine global asymptotic stability of one dimensional population models is given. This is done by relating the enveloping property with the existence of a global Lyapunov function. Following this spirit we revisit a result of Liz.

Keywords: Population models; Enveloping method; Lyapunov function; Global asymptotic stability

1. Introduction

One dimensional population models are described by difference equations of the form

$$x_{n+1} = f(x_n), \quad n \in \mathbb{N}, \tag{1}$$

where $f: (0, x_\infty) \to (0, x_\infty)$, and $0 < x_\infty \le \infty$. It is also assumed that (see Refs. [1,2,4,6]):

- (a_1) f is continuous; has a unique equilibrium at $\bar{x} \in (0, x_\infty)$; $f(x) > x$ for $x < \bar{x}$ and $f(x) < x$ for $x > \bar{x}$.
- (a_2) There exists L such that $f(x) \le L < x_\infty$ for all $0 < x \le \bar{x}$. Observe that when $x_\infty = \infty$ this condition states that f is bounded on $(0, \bar{x}]$ (as in Ref. [4], for instance).

Without loss of generality in the following we take $\bar{x} = 1$.

It has been observed, and in fact proved, in many of the interesting population models of the above type, that the local asymptotic stability of the equilibrium implies the global asymptotic stability of it (LAS $\Rightarrow$ GAS). In some of these interesting models the global stability has been proved by using a method developed by Cull (see Refs. [1] or [2], for instance), which is called *enveloping method*. Although it is more general, we outline their result as follows. According to Cull, for a given population model f it is said that a function ϕ envelops f if $\phi(x) > f(x)$ on (0, 1), and $\phi(x) < f(x)$ for all $x > 1$ (recall that $\bar{x} = 1$ is the equilibrium of f). In practice the enveloping functions need to satisfy decreasing and involutive conditions in a certain interval containing the equilibrium of f in order to guarantee

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