New Advances on the Lyapunov Constants of Some Families of Planar Differential Systems

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Abstract This note presents some advances regarding the Lyapunov constants of some families of planar polynomial differential systems, as a first step toward the resolution of the center and cyclicity problems. First, a parallelization approach is computationally implemented to achieve the 14th Lyapunov constant of the complete cubic family. Second, a technique based on interpolating some specific quantities so as to reconstruct the structure of the Lyapunov constants is used to study a Kukles system, some fifth-degree homogeneous systems, and a quartic system with two invariant lines.

1 Introduction

Let us consider a real polynomial differential system in the plane with some parameters, $\lambda \in \mathbb{R}^d$, written in complex coordinates as

$$\begin{cases} \dot{z} = iz + Z(z, w, \lambda), \\ \dot{w} = -iw + W(z, w, \lambda), \end{cases} \tag{1}$$

where $w = \bar{z}$ and $Z(z, w, \lambda)$, $W(z, w, \lambda) = \bar{Z}(z, w, \lambda)$ are polynomial perturbations having neither linear nor constant terms in z, w. The center problem consists in identifying whether the origin of (1) is a center or a focus, when the origin is a

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