SIAM J. Sci. Comput.
Vol. 28, No. 1, pp. 123–140

A CLASS OF EXTENDED LIMITERS APPLIED TO PIECEWISE HYPERBOLIC METHODS*

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Abstract. Piecewise hyperbolic methods (PHMs) have been widely used for the approximation of hyperbolic conservation laws. In this paper we have studied the "power limiters" in order to analyze their suitable use in the design of PHMs. We formulate a new PHM, based on these limiters, improving the behavior of the PHM in [SIAM J. Sci. Comput., 15 (1994), pp. 892–915], at local extrema, jump discontinuities, and discontinuities in derivative. We show that this new hyperbolic reconstruction (we call it PowerPHM) is local total variation bounded and it behaves stably with a reduced numerical viscosity compared to PHM. We show numerical evidence of the above mentioned features by means of one- and two-dimensional numerical tests with the Euler equations of gas dynamics.

Key words. conservation laws, shock capturing schemes, high order accuracy, piecewise hyperbolic reconstructions, limiters, local total variation bounded

AMS subject classifications. 65M06, 35L65, 76M20

DOI. 10.1137/040611811

1. Introduction.

Our general goal consists in obtaining high order accurate numerical approximations to piecewise smooth real functions of a real variable with a finite number of jump discontinuities to approximate the solution of hyperbolic systems of conservation laws.

Our discussion here will focus on a fixed uniform grid and a fixed degree of local accuracy, trying to minimize the spurious information generated when dealing simultaneously with coarse and fine scales.

A first order approximation to a piecewise smooth function in a fixed grid can be defined as a piecewise constant function, with jump discontinuities located at the cell interfaces [4]. Van Leer considered second order approximations defined as piecewise linear functions, introducing for the first time the minmod limiter function applied on neighboring slopes to avoid the Gibbs phenomena near discontinuities when computing approximate solutions to hyperbolic conservation laws [21, 22, 10]. Total variation diminishing (TVD) schemes were introduced in [6] to get high order accurate nonoscillatory approximations of the solution of hyperbolic conservation laws, but the main drawback of these schemes was the degeneration to first order of accuracy at local extrema [13]. Essentially nonoscillatory (ENO) methods were introduced to get high order accurate reconstruction procedures of order larger than two, avoiding Gibbs phenomena and spurious oscillations, up to local truncation errors [7]. ENO procedures use the smoothest polynomial interpolation by means of choosing the divided differences of smallest size following a tree-like algorithm. The selection procedure of the ENO methods is a limiter function acting on the successive divided differences of the data. One of the important issues of the high order accurate reconstruction procedures is the size of the stencil, i.e., the number of data points needed to perform

*Received by the editors July 16, 2004; accepted for publication (in revised form) August 29, 2005; published electronically March 3, 2006. This work was supported by DGICYT projects BFM2001-2814 and MTM2005-07708.

http://www.siam.org/journals/sisc/28-1/61181.html

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