Numerical Simulation of Relativistic Flows Described by a General Equation of State

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1 Introduction

In this research work we focus on the numerical simulation of the dynamics and thermodynamics of relativistic flows described in terms of the Euler equations of special relativity for fluids whose intrinsic properties are described by means of an equation of state (EOS) for a relativistic perfect gas due to Synge, ([Syn57, FK96]).

The ideal gas EOS is standard in relativistic fluid dynamics ([AIMM99, MIM91, MMFIM97, MM03, NW86]. The use of the ideal gas EOS is computationally convenient but is not the most appropriate EOS to simulate the thermodynamics intrinsic to relativistic flows. The ideal gas EOS forces the gas to keep constant the ratio of specific heats γ along the evolution. In addition, the ideal gas EOS is not consistent with the relativistic formulation of the kinetic theory of gases and allows superluminal speed of sound for fixed adiabatic exponents $\gamma > \frac{5}{3}$.

Synge proposed in 1957 ([Syn57]) the exact form of a relativistic ideal gas EOS to overcome the limitations of the ideal gas EOS. The Synge EOS is given by means of an analytic expression of the relativistic enthalpy as a function of temperature in terms of the Bessel functions of second kind, K_2 and K_3. The use of the Synge EOS allows the gas to reach states thermodynamically relativistic as a direct result of wave dynamics making possible variations of the γ value in a natural way. At the same time, small variations of the γ value produce significant differences on the wave structure along the evolution compared with the case where ideal gas EOS is used.

The main drawback of the Synge EOS is that it is computationally expensive and, as a consequence, it has not been frequently used in numerical simulations of relativistic flows, [FK96].

In this research work we propose a simple and accurate rational approximation to the Synge enthalpy function that is computationally efficient and describes precisely the thermodynamics of relativistic perfect gas. Preliminary