Journal of Computational Physics 228 (2009) 4232–4247

Contents lists available at ScienceDirect

Journal of Computational Physics

journal homepage: www.elsevier.com/locate/jcp

A characteristic-based nonconvex entropy-fix upwind scheme for the ideal magnetohydrodynamic equations ☆

Susana Serna

Department of Mathematics, University of California Los Angeles, Los Angeles, CA 90095-1555, USA

ARTICLE INFO

Article history:
Received 14 April 2008
Received in revised form 3 February 2009
Accepted 2 March 2009
Available online 11 March 2009

Dedicated to Professor Antonio Marquina on the occasion of his 60th birthday.

Keywords:
Hyperbolic conservation laws
Ideal magnetohydrodynamic equations
Local characteristic fields
Upwind schemes

ABSTRACT

In this paper we perform an analysis of the wave structure of the ideal magnetohydrodynamic (MHD) equations. We present an analytical expression of the nonlinearity term associated to each characteristic field derived from a scaled version of the complete system of eigenvectors proposed by Brio and Wu [M. Brio, C.C. Wu, An upwind differencing scheme for the equations of ideal magnetohydrodynamics, J. Comput. Phys. 75 (2) (1988) 400–422] and adopting the eight wave approach by Powell et al. [K.G. Powell, P.L. Roe, R.S. Myong, T. Gombosi, D. deZeeuw, An upwind scheme for magnetohydrodynamics, AIAA 12th Computational Fluid Dynamics Conference, San Diego, CA, 1995, pp. 661–674]. A criterion for the detection of local regions containing points for which a nonlinear characteristic field becomes nonconvex is formulated for the two-dimensional case. We then design a characteristic-based upwind scheme for the ideal MHD equations that resolves the wave dynamics by local characteristic wavefields. The new scheme is able to detect local regions containing nonconvex singularities and to handle an entropy correction through a prescription of a local viscosity ensuring convergence to the entropy solution. A third order accurate version of the scheme performs satisfactorily in resolving one and two-dimensional MHD problems. Numerical results indicate that the proposed scheme behaves low dissipative, stable and accurate under high CFL numbers.

1. Introduction

The ideal magnetohydrodynamics (MHD) system of equations can be expressed as

$$\rho_t = -\nabla(\rho \mathbf{v}), \tag{1}$$

$$(\rho \mathbf{v})_t = -\nabla\left(\rho \mathbf{v}\mathbf{v}^T + \left(P + \frac{1}{2}\mathbf{B}^2\right)I - \mathbf{B}\mathbf{B}^T\right), \tag{2}$$

$$\mathbf{B}_t = \nabla \times (\mathbf{v} \times \mathbf{B}), \tag{3}$$

$$E_t = -\nabla\left(\left(\frac{\gamma}{\gamma - 1}P + \frac{1}{2}\rho q^2\right)\mathbf{v} - (\mathbf{v} \times \mathbf{B}) \times \mathbf{B}\right), \tag{4}$$

where $\rho, P, \mathbf{v}, \mathbf{B}$ and E denote the mass density, the pressure, the velocity field, the magnetic field and the total energy respectively. The adiabatic constant is represented by γ and the energy $E = \frac{1}{2}\rho q^2 + \frac{1}{2}B^2 + \frac{P}{\gamma-1}$ where q^2 and B^2 are the squares of the magnitudes of the velocity field and the magnetic field respectively. The hydrodynamic pressure is defined through the ideal gas EOS as $P = (\gamma - 1)\rho\epsilon$ where ϵ is the specific internal energy.

☆ This work is supported by Grants DGICYT MTM2008-03597 and BAA-H-M1582-07-BA001.
E-mail address: serna@math.ucla.edu

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doi:10.1016/j.jcp.2009.03.001