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Analysis of unstable behavior in a mathematical model for erythropoiesis

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Abstract We consider an age-structured model that describes the regulation of erythropoiesis through the negative feedback loop between erythropoietin and hemoglobin. This model is reduced to a system of two ordinary differential equations with two constant delays for which we show existence of a unique steady state. We determine all instances at which this steady state loses stability via a Hopf bifurcation through a theoretical bifurcation analysis establishing analytical expressions for the scenarios in which they arise. We show examples of supercritical Hopf bifurcations for parameter values estimated according to physiological values for humans found in the literature and present numerical simulations in agreement with the theoretical analysis. We provide a strategy for parameter estimation to match empirical measurements and predict dynamics in experimental settings, and compare existing data on hemoglobin oscillation in rabbits with predictions of our model.

Keywords Age-structured model · Multiple delay differential equations · Hopf bifurcation · Erythropoiesis

Mathematics Subject Classification 35Q92 · 37N25 · 92C50

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