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Fifth-Order Weighted Power-ENO Schemes for Hamilton-Jacobi Equations

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We design a class of Weighted Power-ENO (Essentially Non-Oscillatory) schemes to approximate the viscosity solutions of Hamilton-Jacobi (HJ) equations. The essential idea of the Power-ENO scheme is to use a class of extended limiters to replace the minmod type limiters in the classical third-order ENO schemes so as to improve resolution near kinks where the solution has discontinuous gradients. Then a weighting strategy based on appropriate smoothness indicators lifts the scheme to be fifth-order accurate. In particular, numerical examples indicate that the Weighted Power$_3$ENO5 works for general HJ equations while the Weighted Power$_\infty$ENO5 works for non-linear convex HJ equations. Numerical experiments also demonstrate the accuracy and the robustness of the new schemes.

KEY WORDS: Hamilton-Jacobi; ENO; Weighted Power-ENO; level set; monotone schemes.

1. INTRODUCTION

We consider the initial value problem for the Hamilton-Jacobi equation

$$\phi_t + H(x, \phi, \nabla\phi) = 0, \quad \phi(x, 0) = \phi_0(x), \quad x \in R^d, \ t > 0, \tag{1.1}$$

where H is a non-decreasing function of ϕ.

Such Hamilton-Jacobi (HJ) equations appear in many applications, for example, geometrical optics, optimal control, differential games, material sciences and calculus of variations. Therefore, it is essential to develop efficient, high-order accurate numerical methods to solve these equations.

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