Journal of Computational Mathematics
Vol.28, No.4, 2010, 552–568.

http://www.global-sci.org/jcm
doi:10.4208/jcm.1003-m0016

A STOPPING CRITERION FOR HIGHER-ORDER SWEEPING SCHEMES FOR STATIC HAMILTON-JACOBI EQUATIONS*

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Abstract

We propose an effective stopping criterion for higher-order fast sweeping schemes for static Hamilton-Jacobi equations based on ratios of three consecutive iterations. To design the new stopping criterion we analyze the convergence of the first-order Lax-Friedrichs sweeping scheme by using the theory of nonlinear iteration. In addition, we propose a fifth-order Weighted PowerENO sweeping scheme for static Hamilton-Jacobi equations with convex Hamiltonians and present numerical examples that validate the effectiveness of the new stopping criterion.

Mathematics subject classification: 65N06, 65N12, 35F21
Key words: Fast sweeping methods, Gauss-Seidel iteration, High order accuracy, Static Hamilton-Jacobi equations, Eikonal equations.

1. Introduction

Consider the following static Hamilton-Jacobi (H-J) equation:

$$\begin{cases} H(\nabla\phi(x), x) = f(x), & x \in \Omega \setminus \Gamma, \\ \phi(x) = g(x), & x \in \Gamma \subset \Omega, \end{cases} \tag{1.1}$$

where $g(x)$ is a positive, Lipschitz continuous function, Ω is an open, bounded polygonal domain in R^d, and Γ is a subset of Ω. $H(p, x)$ is Lipschitz continuous in both arguments, and it is convex and homogeneous of degree one in the first argument.

This class of first-order nonlinear PDEs arise in many applications such as optimal control, differential games, computer vision, geometric optics, and geophysical applications. Thus it is essential to develop efficient high-order accurate numerical methods for such equations. Based on [22, 27] we propose a fifth-order sweeping scheme for the equation. To design an effective stopping criterion for the sweeping scheme, we analyze convergence of the first-order Lax-Friedrichs scheme in terms of theory of nonlinear iterative methods.

Fast sweeping methods are a family of efficient methods for solving static Hamilton-Jacobi equations [3, 7–9, 11, 18, 19, 25, 28, 29], and some essential ideas of these methods may trace

* Received November 29, 2008 / Revised version received May 3, 2009 / Accepted October 17, 2009 / Published online April 19, 2010 /