Characterization of Transversal Homothetic Solutions in the n-body Problem

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Abstract

Homothetic solutions of the n-body problem can be seen as heteroclinic orbits when the dynamical variables are changed via the McGehee blow-up and the time is suitably scaled. Transversality of the invariant asymptotic manifolds which contain the heteroclinic orbits is related to some structural stability. We fully characterize the cases in which such transversality is obtained for the n-body problem in any dimension.

§ 1. Introduction

The classical n-body problem consists in discribing the motion of n punctual masses under their mutual gravitational attraction in ν-dimensional Euclidean space. By "configuration" we understand the simultaneous position of the n bodies. A configuration is called central [11] if the acceleration of any one of the bodies is proportional to its position with respect to the center of mass and the constant of proportionality is independent of the body. A solution of the n-body problem such that the passage from the configuration at time t_0 to the configuration at time t is a homothecy is called a homothetic solution or homothetic orbit. The configuration corresponding to a homothetic orbit is central at every time when it exists.

The set of central configurations is invariant under the group of similitudes. Two central configurations are equivalent if there is a similitude that maps the one into the other. There is only one class of equivalence if $n = 2$. For $n = 3$ there are three classes if $\nu = 1$. Otherwise there are five classes. It is an open question whether the number of classes is finite when $n > 3$. However in the space of masses the set of points for which the number of classes is not finite has zero Lebesgue measure. For more details and motivation see SMALE [9].

We study the homothetic orbits of the n-body problem in a ν-dimensional space. Such orbits begin in a total ejection, *i.e.* there is some time t_e such that as