ANALYTIC CONTINUATION OF CIRCULAR AND ELLIPTIC KEPLER MOTION TO THE GENERAL 3–BODY PROBLEM

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The existence of families of periodic solutions of the general planetary 3–body problem in which one of the orbits is ellipic is shown. Each family derives from uncoupled circular and elliptic Kepler motion in a given resonance, with the eccentricity of the elliptic orbit as the parameter. The standard method of Poincaré's analytic continuation cannot be directly applied due to the vanishing of a determinant, so a strong form of the implicit function theorem is used.

1 Introduction

Let P_0, P_1 and P_2 be three material points of mass m_0, m_1 and m_2, respectively, moving in a plane under their mutual Newtonian gravitational attraction. We are particularly interested in the *planetary problem*, in which one of the bodies is much larger than the other ones. As m_0 will be asumed to be the large mass then we will call P_0 *the Sun* and P_1 and P_2 *the planets.* We will set $m_0 = 1$, $m_1 = \nu_1\mu, m_2 = \nu_2\mu$, where μ is a small parameter. We are interested in periodic solutions which, in the limit $\mu \to 0$, become circular and elliptic resonant orbits of two uncoupled Kepler problems. These periodic solutions will be shown to exist, when $\mu \neq 0$ provided a certain integral does not vanish. This integral is computed numerically in a few instances and shown to be different from zero.

2 The equations of motion

We consider an inertial frame with origin at the center of mass of P_0, P_1 and P_2, denote by $q_i = (q_{i1}, q_{i2})$, $i = 0, 1, 2$, the position of P_i in this frame and by $\dot{q}_i = (\dot{q}_{i1}, \dot{q}_{i2})$ its velocity. The Lagrangian of the system is then given by

$$\begin{aligned}\mathcal{L}_0 = {} & \frac{m_0}{2}||\dot{q}_0||^2 + \frac{m_1}{2}||\dot{q}_1||^2 + \frac{m_2}{2}||\dot{q}_2||^2 \\ & + \frac{m_0 m_1}{||q_0 - q_1||} + \frac{m_0 m_2}{||q_0 - q_2||} + \frac{m_1 m_2}{||q_1 - q_2||}.\end{aligned} \tag{1}$$