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Heteroclinic, Homoclinic and Closed Orbits in the Chen System

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Bounded orbits such as closed, homoclinic and heteroclinic orbits are discussed in this work for a Lorenz-like 3D nonlinear system. For a large spectrum of the parameters, the system has neither closed nor homoclinic orbits but has exactly two heteroclinic orbits, while under other constraints the system has symmetrical homoclinic orbits.

Keywords: ODE systems; bifurcations; homoclinic and heteroclinic orbits.

1. Introduction

In this paper, we consider a Lorenz-like three-dimensional system [Lorenz, 1963], namely the Chen system. Interesting results reported recently on this system concerning closed, homoclinic and heteroclinic orbits are found in [Li *et al.*, 2006]. More results on the Chen system are given in [Chang & Chen, 2006; Li *et al.*, 2009; Li & Wang, 2013; Zhou *et al.*, 2004; Llibre *et al.*, 2012; Llibre & Rodrigues, 2015; Lu & Zhang, 2007] and in some references therein. In this paper, we first refine some results reported in [Li *et al.*, 2006] and present a slightly different proof of these results. Secondly, we consider an important case not treated in this article and show the system has homoclinic orbits for a large spectrum of the parameters, by transforming the system into a new form and using results reported in [Belykh, 1984]. Proving the existence of homoclinic or heteroclinic orbits in nonlinear ODE systems is in general a difficult task. Some methods dealing with such orbits can be found in [Cao *et al.*, 2011; Lazureanu & Binzar, 2012a, 2012b; Tigan & Constantinescu, 2009].

The Chen system is given by:

$$\begin{aligned} \dot{x} &= a(y - x), \\ \dot{y} &= (c - a)x - xz + cy, \\ \dot{z} &= xy - bz, \end{aligned} \tag{1}$$

where $a > 0$, $b > 0$ and $c > 0$ are positive real parameters.

Recall that, if x_0 is a hyperbolic equilibrium point such that the stable manifold $W^s(x_0)$ intersected with the unstable manifold $W^u(x_0)$ is not empty, then the orbits belonging to $W^s(x_0) \cap W^u(x_0) \neq \Phi$ are called *homoclinic* orbits. They are doubly asymptotic to the equilibrium point x_0. Similarly, if x^1, x^2 are two hyperbolic equilibrium points such that there exists an orbit $\Gamma \subset W^s(x^1) \cap W^u(x^2)$ or inversely $\Gamma \subset W^u(x^1) \cap W^s(x^2)$, then Γ is called a *heteroclinic* orbit.

The system (1) has the origin O as an equilibrium point for any $a, b, c > 0$ and it has two more equilibrium points

$$S_1 = (\sqrt{b(2c-a)}, \sqrt{b(2c-a)}, 2c - a)$$