Chapter

On Dynamics and Invariant Sets in Predator-Prey Maps

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Abstract

A multitude of physical, chemical, or biological systems evolving in discrete time can be modelled and studied using difference equations (or iterative maps). Here we discuss local and global dynamics for a predator-prey two-dimensional map. The system displays an enormous richness of dynamics including extinctions, co-extinctions, and both ordered and chaotic coexistence. Interestingly, for some regions we have found the so-called hyperchaos, here given by two positive Lyapunov exponents. An important feature of biological dynamical systems, especially in discrete time, is to know where the dynamics lives and asymptotically remains within the phase space, that is, which is the invariant set and how it evolves under parameter changes. We found that the invariant set for the predator-prey map is very sensitive to parameters, involving the presence of escaping regions for which the orbits go out of the domain of the system (the species overcome the carrying capacity) and then go to extinction in a very fast manner. This theoretical finding suggests a potential dynamical fragility by which unexpected and sharp extinctions may take place.

Keywords: bifurcations, chaos, invariant sets, maps, nonlinearity, ecology

1. Introduction

Natural and artificial complex systems can evolve in discrete time, often resulting in extremely complex dynamics such as chaos. A well-known example of such a complexity is found in ecology, where discrete-time dynamics given by a yearly climatic forcing can make the population emerging a given year to be a discrete function of the population of the previous one [1]. Although early work already pointed towards complex population fluctuations as an expected outcome of the nonlinear nature of species interactions [2], the first evidence of chaos in species dynamics was not characterised until the late 1980s and 1990s [3, 4]. Since pioneering works on one-dimensional maps [5, 6], the field of dynamical complexity in ecology experienced a rapid development [5–7], with several key investigations offering a compelling evidence of chaotic dynamics in insect species in nature [1, 3, 4].

Discrete-time models have played a key role in the understanding of complex ecosystems, especially for univoltine species (i.e. species undergoing one generation per year) [5, 6]. Many insects inhabiting temperate and boreal climatic zones