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ON THE REVERSIBLE QUADRATIC CENTERS WITH MONOTONIC PERIOD FUNCTION

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ABSTRACT. This paper is devoted to studying the period function of the quadratic reversible centers. In this context the interesting stratum is the family of the so-called Loud's dehomogenized systems, namely

$$\begin{cases} \dot{x} = -y + xy, \\ \dot{y} = x + Dx^2 + Fy^2. \end{cases}$$

We determine several regions in the parameter plane for which the corresponding center has a monotonic period function. To this end we first show that any of these systems can be brought by means of a coordinate transformation to a potential system. Then we apply a monotonicity criterium of R. Schaaf.

1. INTRODUCTION AND STATEMENT OF THE RESULT

The present paper deals with the period function of the quadratic centers. The literature has used different terminology to classify these centers but essentially there are four families: Hamiltonian, reversible Q_3^R, codimension four Q_4 and generalized Lotka-Volterra systems Q_3^{LV}. According to Chicone's conjecture [1] the reversible centers have at most two critical periods, and the centers of the three other families have a monotonic period function. In fact there is much analytic evidence that the conjecture is true. Indeed, Coppel and Gavrilov [7] proved that the period function of any Hamiltonian quadratic center is monotonous and, more recently, Zhao [23] showed that the codimension four centers have the same property. Concerning the Q_3^{LV} centers there are very few results. In the middle 80s several authors [13, 16, 21] showed independently the monotonicity of the classical Lotka-Volterra centers (which constitute a hypersurface inside the Q_3^{LV} family), and more recently the same property has been proved in [20] for two other hypersurfaces.

From the point of view of the study of the period function it is clear therefore that the most interesting family of centers is the reversible one. By an affine transformation and a constant rescaling of time (see [26] for instance), any reversible quadratic center can be brought to *Loud's normal form*

$$\begin{cases} \dot{x} = -y + Bxy, \\ \dot{y} = x + Dx^2 + Fy^2. \end{cases}$$

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