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The period function of the generalized Lotka–Volterra centers

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Abstract

The present paper deals with the period function of the quadratic centers. In the literature different terminologies are used to classify these centers, but essentially there are four families: Hamiltonian, reversible Q_3^R, codimension four Q_4 and generalized Lotka–Volterra systems Q_3^{LV}. Chicone [C. Chicone, Review in MathSciNet, Ref. 94h:58072] conjectured that the reversible centers have at most two critical periods, and that the centers of the three other families have a monotonic period function. With regard to the second part of this conjecture, only the monotonicity of the Hamiltonian and Q_4 families [W.A. Coppel, L. Gavrilov, The period function of a Hamiltonian quadratic system, Differential Integral Equations 6 (1993) 1357–1365; Y. Zhao, The monotonicity of period function for codimension four quadratic system Q_4, J. Differential Equations 185 (2002) 370–387] has been proved. Concerning the Q_3^{LV} family, no substantial progress has been made since the middle 80s, when several authors showed independently the monotonicity of the classical Lotka–Volterra centers [F. Rothe, The periods of the Volterra–Lokta system, J. Reine Angew. Math. 355 (1985) 129–138; R. Schaaf, Global behaviour of solution branches for some Neumann problems depending on one or several parameters, J. Reine Angew. Math. 346 (1984) 1–31; J. Waldvogel, The period in the Lotka–Volterra system is monotonic, J. Math. Anal. Appl. 114 (1986) 178–184]. By means of the first period constant one can easily conclude that the period function of the centers in the Q_3^{LV} family is monotone increasing near the inner boundary of its period annulus (i.e., the center itself). Thus, according to Chicone's conjecture, it should be also monotone increasing near the outer boundary, which in the Poincaré disc is a polycycle. In this paper we show that this is true. In addition we prove that, except for a zero measure subset of the parameter plane, there is no bifurcation of critical periods from the outer boundary. Finally we show that the period function is globally (i.e., in the whole period annulus) monotone increasing in two other cases different from the classical one.

Keywords: Center; Period function; Critical period; Bifurcation

1. Setting of the problem and results

This paper is concerned with the period function of centers. A critical point p of a planar differential system is a *center* if it has a punctured neighbourhood that consists entirely of periodic orbits surrounding p. The largest punctured neighbourhood with this property is called the *period annulus* of the center and, in what follows, it will be denoted by $\mathcal{P}$. The *period function* of the center assigns to each periodic orbit in $\mathcal{P}$ its period. Questions related

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