Applied Mathematics and Computation 216 (2010) 1956–1964

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

# On the period function in a class of generalized Lotka–Volterra systems

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ARTICLE INFO

*Keywords:*
Lotka–Volterra model
Center
Period function
Critical period

ABSTRACT

In this note, motivated by the recent results of Wang et al. [Wang et al., Local bifurcations of critical periods in a generalized 2D LV system, Appl. Math. Comput. 214 (2009) 17–25], we study the behaviour of the period function of the center at the point (1,1) of the planar differential system

$$\begin{cases} u' = u^p(1 - v^q), \\ v' = \mu v^q(u^p - 1), \end{cases}$$

where $p, q, \mu \in \mathbb{R}$ with $pq > 0$ and $\mu > 0$. Our aim is twofold. Firstly, we determine regions in the parameter space for which the corresponding system has a center with a monotonic period function. Secondly, by taking advantage of the results of Wang et al., we show some properties of the bifurcation diagram of the period function and we make some comments for further research. The differential system under consideration is a generalization proposed by Farkas and Noszticzius of the Lotka–Volterra model.



## 1. Introduction and statement of the result

Simple nonlinear dynamic models with periodic solutions have a great importance in describing complex dissipative systems, especially in chemistry and biology. The Lotka–Volterra model is a classical example and it is widely used to describe chemical and biological systems that oscillate.

The generalization of the Lotka–Volterra model proposed by Farkas and Noszticzius in [9] contains three chemical reactions: **(GLV1)** autocatalytic production of the intermediate $X$, **(GLV2)** autocatalytic transformation of $X$ into $Y$, and **(GLV3)** decay of the intermediate $Y$. It can be formulated by the following reaction schemes:

$$\begin{aligned} &A + pX \xrightarrow{k_1} (p+1)X, && (\textbf{GLV1}), \\ &pX + qY \xrightarrow{k_2} (q+1)Y, && (\textbf{GLV2}), \\ &qY \xrightarrow{k_3} B, && (\textbf{GLV3}), \end{aligned}$$

where $X$ and $Y$ are the intermediates, $A$ and $B$ are reactants of constant concentration, $p$ and $q$ are respectively, the molecules of $X$ and $Y$ that participate in one elementary reaction, and $k_i$ (for $i = 1, 2, 3$) is the reaction rate of the $i$th step. By simple law of mass action principles, the kinetics of these schemes can be described by the following system of ordinary differential equations:

$$\begin{cases} \dot{x} = k_1 a x^p - p k_2 x^p y^q, \\ \dot{y} = k_2 x^p y^q - q k_3 y^q, \end{cases} \tag{1}$$

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0096-3003/$ - see front matter © 2010 Elsevier Inc. All rights reserved.
doi:10.1016/j.amc.2010.03.025